Critical endpoint of QCD at finite T and μ, lattice results

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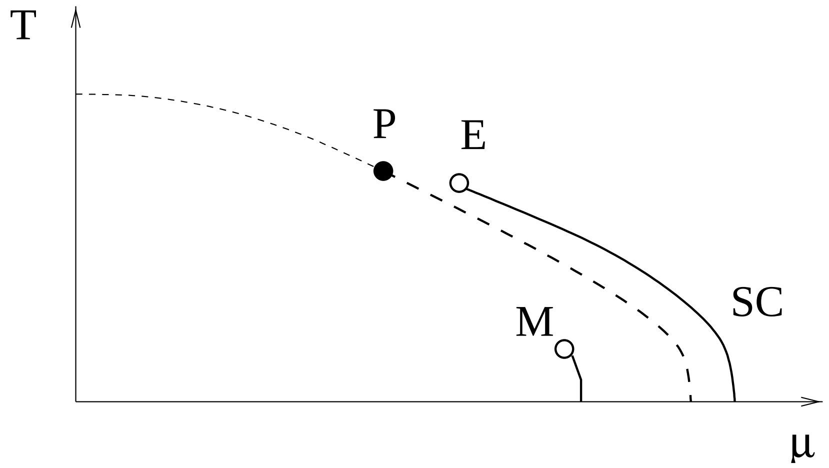
Introduction, experimental motivation



- Chiral phase transition (PT)

$n_f = 2$ with $m_q = 0$ at $\mu = 0 \Rightarrow 2^{nd}$ order PT
$n_f = 2$ with $m_q = 0$ at $T = 0 \Rightarrow 1^{st}$ order PT
$n_f = 2$ with $m_q = 0 \Rightarrow$ tricritical point (P) at μ,T$\neq$0

$n_f = 3$ with $m_q = 0$ at $\mu = 0 \Rightarrow 1^{st}$ order PT
increasing m_s weakens the 1^{st} order PT $\Rightarrow$ cross-over

$n_f = 2 + 1$ with physical m_q at $\mu = 0 \Rightarrow$ cross-over
$n_f = 2 + 1$ with physical m_q at $T = 0 \Rightarrow 1^{st}$ order PT
$n_f = 2 + 1$ with physical $m_q \Rightarrow$ critical endpoint (E) at μ,T$\neq$0

"If and when the critical point E is discovered, it will appear prominently on the map of the phase diagram featured in any future textbook of QCD."

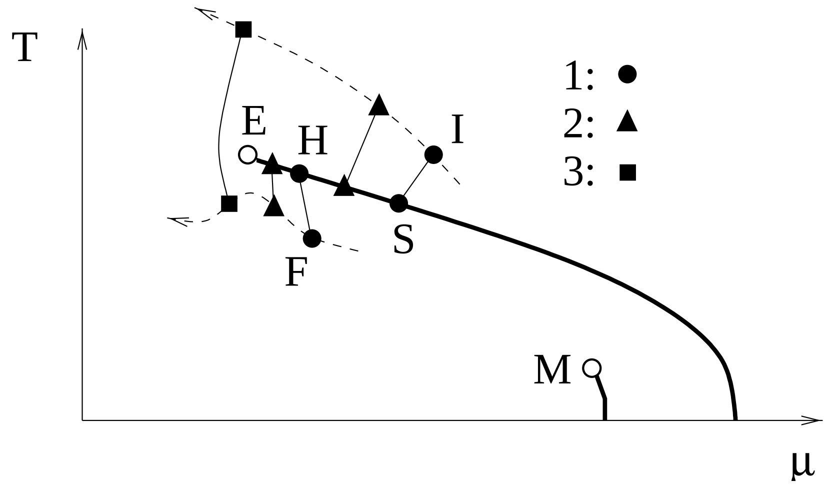
- locate the endpoint: nonperturbative prediction of QCD

lattice gauge theory: serious problems at $\mu \neq 0$
functional measure (Euclidean Dirac operator's determinant) complex $\Rightarrow$ spoils any importance sampling method

I.M. Barbour et al., Nucl. Phys. B (Proc. Supl.) 60A, 220 (1998)
Glasgow method: μ reweighting based on an ensemble at $\mu = 0$
after collecting 20 million configurations only unphysical results
$T = \mu = 0$ ensemble does not overlap with the transition states

M.A. Halasz et al., Phys. Rev. D58, 096007 (1998)
random matrix model for the Dirac operator can be solved
crude estimate, prediction can be off by a factor of 2-3
$\Rightarrow T_E \approx 120$ MeV and $\mu_E \approx 700$ MeV

M. Stephanov, K. Rajagopal, E. Shuryak, Phys. Rev. Lett., 81, 4816 (1998)



• control parameters to decrease μ:
increasing the energy of the collision
increasing the centrality of the collision
slicing each event in (smaller) rapidities

zigzag trajectories: latent heat reheats the mixed phase
focusing the trajectories towards E (H is closer than S)
no fine-tuning is needed to explore the singular E point

excess in the low p_T pion spectra
non-monotonic behaviour: E can be missed on either sides

Overlap ensuring multi-parameter reweighting

- generic system with ψ fermions and ϕ bosons

fermionic Lagrangian: $\bar{\psi} M(\phi)\psi \Rightarrow$ after Grasmann integration

$$Z(\alpha) = \int \mathcal{D}\phi \, \mathrm{exp}[-S_{bos}(\alpha,\phi)] \, \mathrm{det}\, M(\phi,\alpha)$$

α: parameter set (gauge coupling, mass, chemical potential)

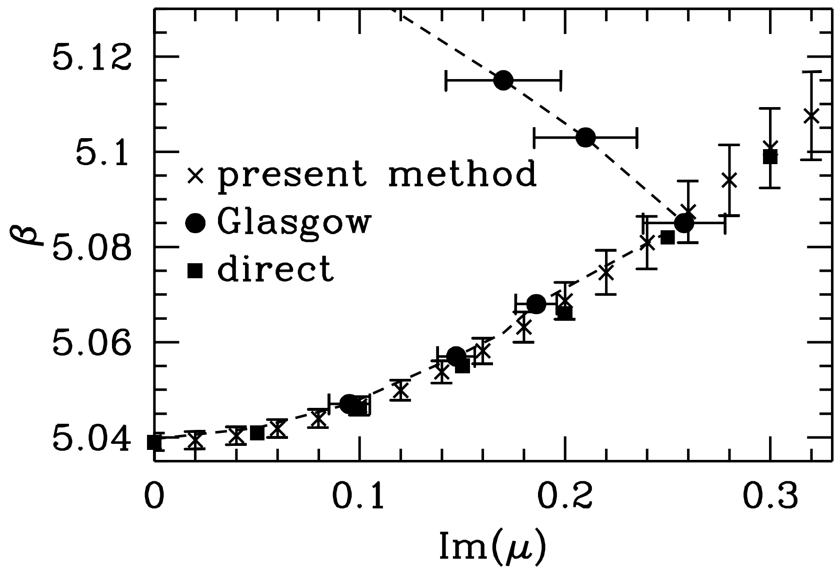
include μ: forward/backward links multiplied with exp($\pm\mu$)
for some parameters α_0 importance sampling can be done

$$Z(\alpha)/Z(\alpha_0) = \int \mathcal{D}\phi \, \mathrm{exp}[-S_{bos}(\alpha_0,\phi)] \, \mathrm{det}\, M(\phi,\alpha_0)$$
$$\{\mathrm{exp}[-S_{bos}(\alpha,\phi) + S_{bos}(\alpha_0,\phi)] \, \mathrm{det}\, M(\phi,\alpha)/ \, \mathrm{det}\, M(\phi,\alpha_0)\}$$

first line: measure; curly bracket: observable (will be measured)
simultaneously changing several parameters: better overlap
e.g. transition configurations are mapped to transition ones

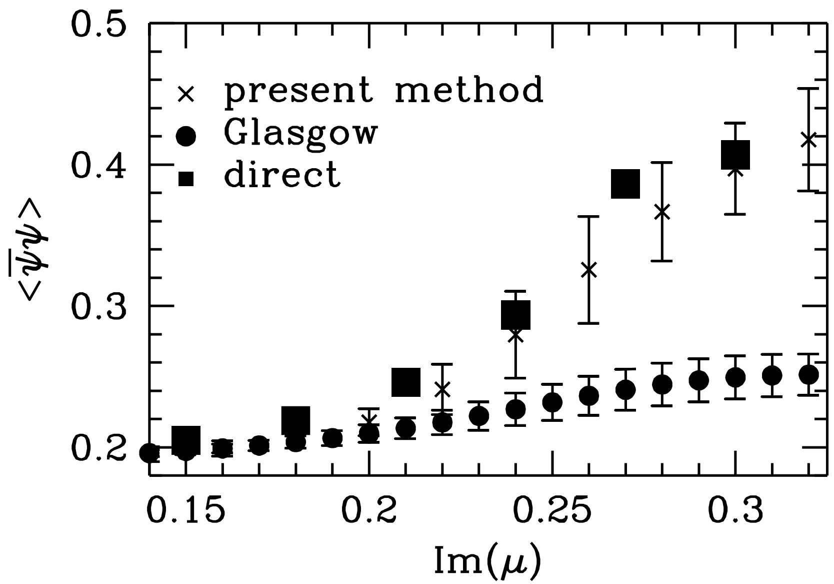
• direct test: n_f=4 dynamical QCD with imaginary μ
m_q=0.05 staggered fermions on $4 \cdot 6^3$ lattices

• compare: direct method and different types of reweighting:
our method: measuring the determinants and bosonic action
Glasgow method: only determinants
phase diagram in the Im(μ)-β plane



overlap is much better for our method
no premature (Glasgow type) onset transition
no “fake” transitions (with correct μ-β)

- chiral condensate at $\mu \approx 0.25$ for the three methods



no "fake" transitions (with correct μ-β)
Glasgow-method: based on an ensemble of the high-T phase
high-μ phase does not overlap with the states of interest
statistical errors: two-parameter reweighting more trustworthy

transition points can be defined by susceptibility peaks,
turning points, real part of Lee-Yang zeros (see later)

we expect that our method is also superior at Re(μ)$\neq$0

QCD with n_f=2+1 dynamical staggered fermions

- partition function with multi-parameter reweighting

$$Z(\alpha)/Z(\alpha_0) = \int \mathcal{D}\phi \exp[-S_{bos}(\alpha_0,\phi)][\det M(\phi,\alpha_0)]^{n_f/4}$$
$$\{\exp[-S_{bos}(\alpha,\phi) + S_{bos}(\alpha_0,\phi)][\det M(\phi,\alpha)]^{n_f/4}/[\det M(\phi,\alpha_0)]^{n_f/4}\}$$

we measure fractional powers of the complex determinants
$\Rightarrow$ choose among the possible Riemann-leaves

a. gauge fix to $A_0 = 0$ on all but the last timeslice
b. multiply the j-th row/column by $e^{\pm j\mu}$
c. rearrange the columns of the matrix
d. L_t-2 Gauss elimination steps gives a $6L_s^3 \times 6L_s^3$ matrix

$$\det M(\mu) = e^{-3L_sL_t\mu} \prod_{i=1}^{6L_s^3} (e^{L_t\mu} - \lambda_i)$$

$\Rightarrow$ gives Z for "arbitrary" μ and β

Lee-Yang zeros of the partition function

C.N. Yang and T.D. Lee, Phys. Rev. 87, 404 (1952)

- distinguish between a crossover and a 1^{st} order PT

1^{st} order PT: free energy $\propto \log Z(\beta)$ is singular
at that point the partition function is zero
PT appears not at finite V, but only at V$\to \infty$
Z has zeros even at finite V, at complex parameters (β)
Re(β_0), zero with smallest imaginary part: transition point

for 1^{st} order PT: zeros approach the real axis
$1/V$ scaling in the V$\to \infty$ limit
generates the singularity of the free energy

crossover: no singularity, zeros do not approach the real axis

Lee-Yang technique: electroweak phase transition's endpoint
F. Csikor, Z. Fodor and J. Heitger, Phys. Rev. Lett. 82, 21 (1999)

Endpoint of 2+1 flavor QCD on $L_t = 4$ lattices

• three basic steps of the analysis

m_s=0.2 (approx. physical), m_{ud}=0.025 (≈3-4× heavier)

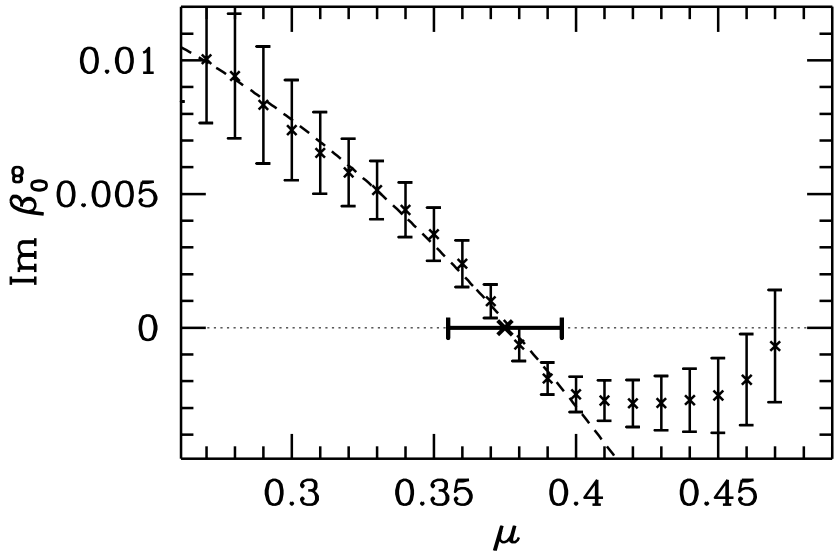
a. determine the transition points, Re(β_0), on L_s=4,6,8 lattices as a function of μ by the Lee-Yang zeros for $\mu \neq 0$ overlap ensuring multi-parameter reweighting (14000, 3600 and 840 independent configurations, respectively)

b. by inspecting the $V \to \infty$ limit of Im(β_0) separate the crossover and the 1^{st} order PT regions in μ

c. connect μ=T=0 lattice parameters with observables: physical scale by R_0 (1/403 MeV), m_ρ (770 MeV), $\sqrt{\sigma}$ (440 MeV) (2×220 independent configurations on $10^3 \cdot 16$ lattices)

• separate the crossover and the 1^{st} order PT

$V \to \infty$ limit of Im(β_0) as a function of μ



small μ: Im(β_0^∞) inconsistent with 0 $\Rightarrow$ crossover

increasing μ: Im(β_0^∞) decreases $\Rightarrow$

transition becomes consistent with a 1^{st} order PT

errors decrease close to E, and Im(β_0^∞) slightly overshoots:

known effects of the Lee-Yang technique for relatively small V-s

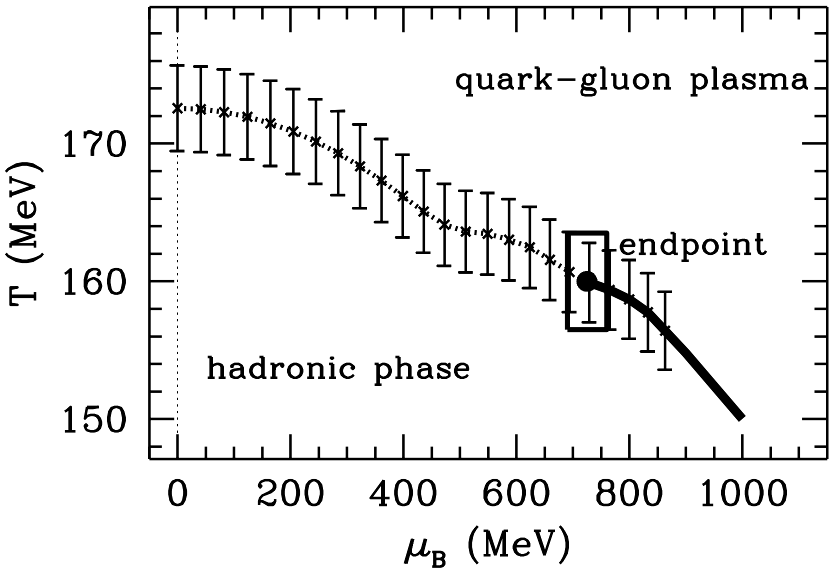
endpoint chemical potential: $\mu_{end} = 0.375(20)$

• phase diagram in physical units

results at T=0 with $R_0 \cdot m_\pi$=0.73(6) (twice too much)

β	m_π	m_ρ	R_0	$\sqrt{\sigma}$
5.208	0.393(2)	1.22(2)	1.87(3)	0.58(7)
5.164	0.393(2)	1.28(3)	1.76(5)	0.75(5)

• T as a function of the baryonic chemical potential μ_B



• lattice result for QCD with n_f=2+1 fermions and $L_t = 4$

endpoint: $T_E = 160 \pm 3.5$ MeV, $\mu_E = 725 \pm 35$ MeV

at μ_B=0 transition temperature: $T_c = 172 \pm 3$ MeV.

Summary, outlook

• critical endpoint in the μ-T plane: unambiguous, non-perturbative prediction of the QCD Lagrangian $\Rightarrow$ important experimental consequences for heavy ion collisions

• lattice QCD at finite μ is an old, unsolved problem
new method: overlap ensuring multi-parameter reweighting presumably good enough to locate the above endpoint (though, can not describe the color superconducting phase)

• overlap ensuring multi-parameter reweighting:
standard importance sampling with reweighting in β and μ maps transition configurations to transition ones (or hadronic/QGP configurations to hadronic/QGP ones)

• can be applied to any number of Wilson or staggered quarks
analytic expression for the determinants for any μ

• direct test for n_f=4 dynamical QCD at imaginary μ
our method: complete agreement with direct results
Glasgow method: premature onset, "fake" transitions

• T=0 and T≠0 simulations in QCD with n_f=2+1 quarks
infinite volume behavior of the Lee-Yang zeros
tells the difference between a 1^{st} order PT and a crossover

endpoint: $T_E = 160 \pm 3.5$ MeV, $\mu_E = 725 \pm 35$ MeV
at μ_B=0 transition temperature: $T_c = 172 \pm 3$ MeV.

• future plans: approaching the chiral and continuum limits

for our m_{ud} production and reweighting need similar CPU-times
for physical masses reweighting in μ is subdominant
evaluating determinants/eigenvalues $\propto L_s^9$
configuration production is at least $\propto L_s^9$
$\Rightarrow$ reweighting in μ remains subdominant