PROGRESS IN QCD NLO JET CALCULATIONS

Zoltán Trócsányi

Institute of Nuclear Research and University of Debrecen, Hungary

Budapest, July 12, 2001.

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  - available techniques and programs

Parallel session 1, abstracts: #606, #882

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- Introduction
  - jet cross sections at NLO
  - available techniques and programs
- Three-jet production in DIS at NLO
- The dipole method for massive partons
- Summary and outlook

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JET CROSS SECTIONS AT NLO

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General solution:

$$\sigma^{\mathrm{NLO}} = \int_{m+1} \left[\left(\mathrm{d}\sigma^{\mathrm{R}}\right)_{\varepsilon=0} - \left(\mathrm{d}\sigma^{\mathrm{A}}\right)_{\varepsilon=0} \right] + \int_m \left[\mathrm{d}\sigma^{\mathrm{V}} + \int_1 \mathrm{d}\sigma^{\mathrm{A}} \right]_{\varepsilon=0}$$

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vi) can be implemented in a fully process independent way.

NLO PROGRAMS FOR JET PRODUCTION

Jet hadroproduction

- W. Giele, E.W.N. Glover, D.A. Kosower (JETRAD): one- and two-jet observables, slicing with invariant-mass cut
- S.D. Ellis, Z. Kunszt and D.E. Soper: one- and two-jet observables, residue
- S. Frixione: one- and two-jet observables, residue
- W. Kilgore and W.T. Giele: three-jet observables, slicing with invariant-mass cut

NLO PROGRAMS FOR JET PRODUCTION

Jet photoproduction

- B.W. Harris and J.F. Owens: one- and two-jet observables, slicing with energy and invariant-mass cuts (two-cut method)
- M. Klasen and G. Kramer: one- and two-jet observables, slicing with invariant-mass cut
- S. Frixione: one- and two-jet observables, residue
- P. Aurenche, L. Bourhis, M. Fontannaz, J.P. Guillet: one- and two-jet observables, slicing with E_T, η and ϕ cuts

NLO PROGRAMS FOR JET PRODUCTION

Jet leptoproduction

- S. Catani and M.H. Seymour (DISENT): structure functions, two-jet observables, dipole
- D. Graudenz (DISASTER++): structure functions, two-jet observables, residue
- Z. Nagy and Z. T. (NLOJET++): structure functions, two- and three-jet observables, dipole (new)

THREE-JET LEPTOPRODUCTION AT NLO

Kinematic region of H1:

$$5\,\mathrm{GeV}^2 < Q^2 < 5000\,\mathrm{GeV}^2 \qquad 0 < x_{\mathrm{Bj}} < 1 \qquad 0.2 < y < 0.6$$

$$-1 < \eta_{\mathrm{jet}}^{\mathrm{lab}} < 2.5 \qquad E_{T,\mathrm{jet}}^{\mathrm{Breit}} > 5\,\mathrm{GeV}$$

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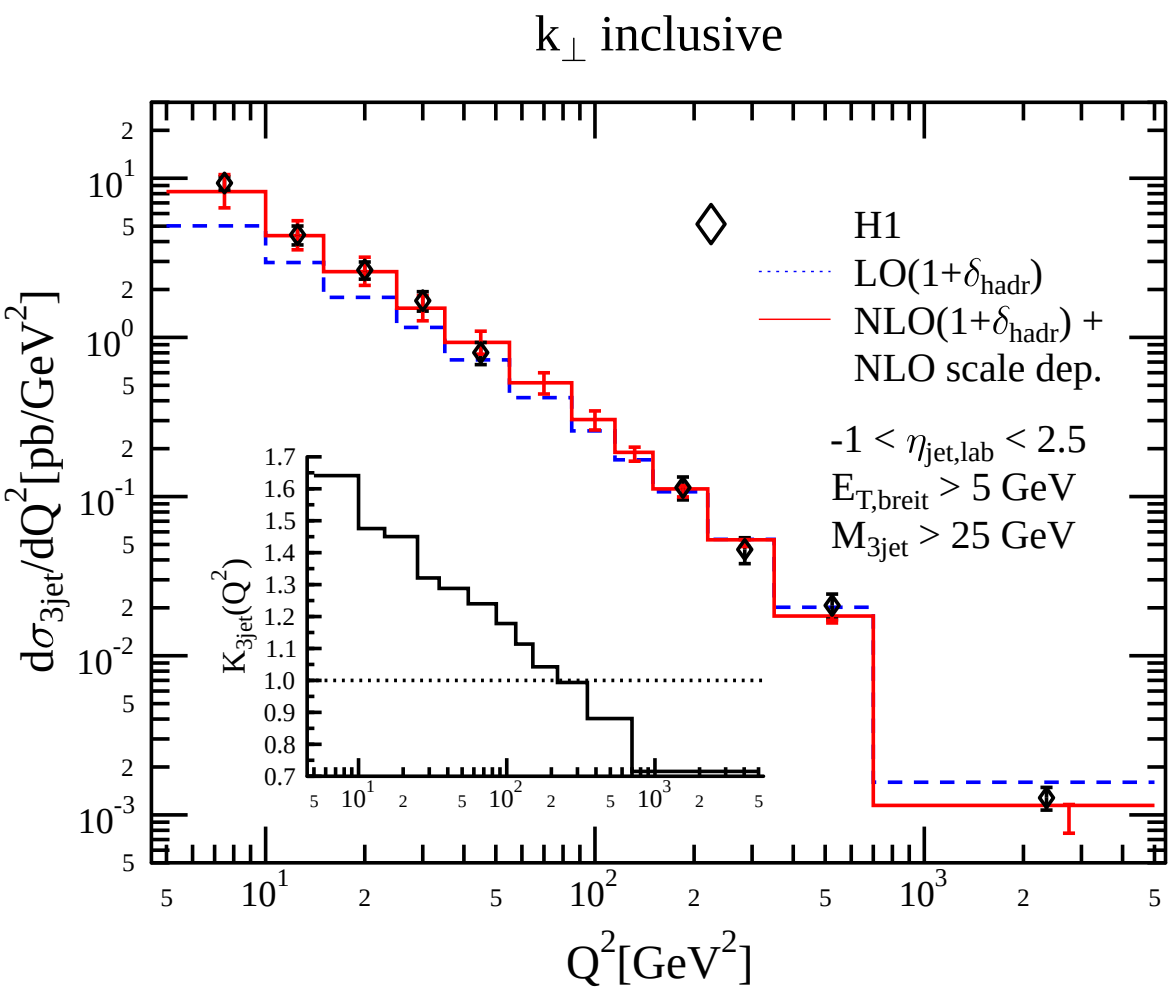
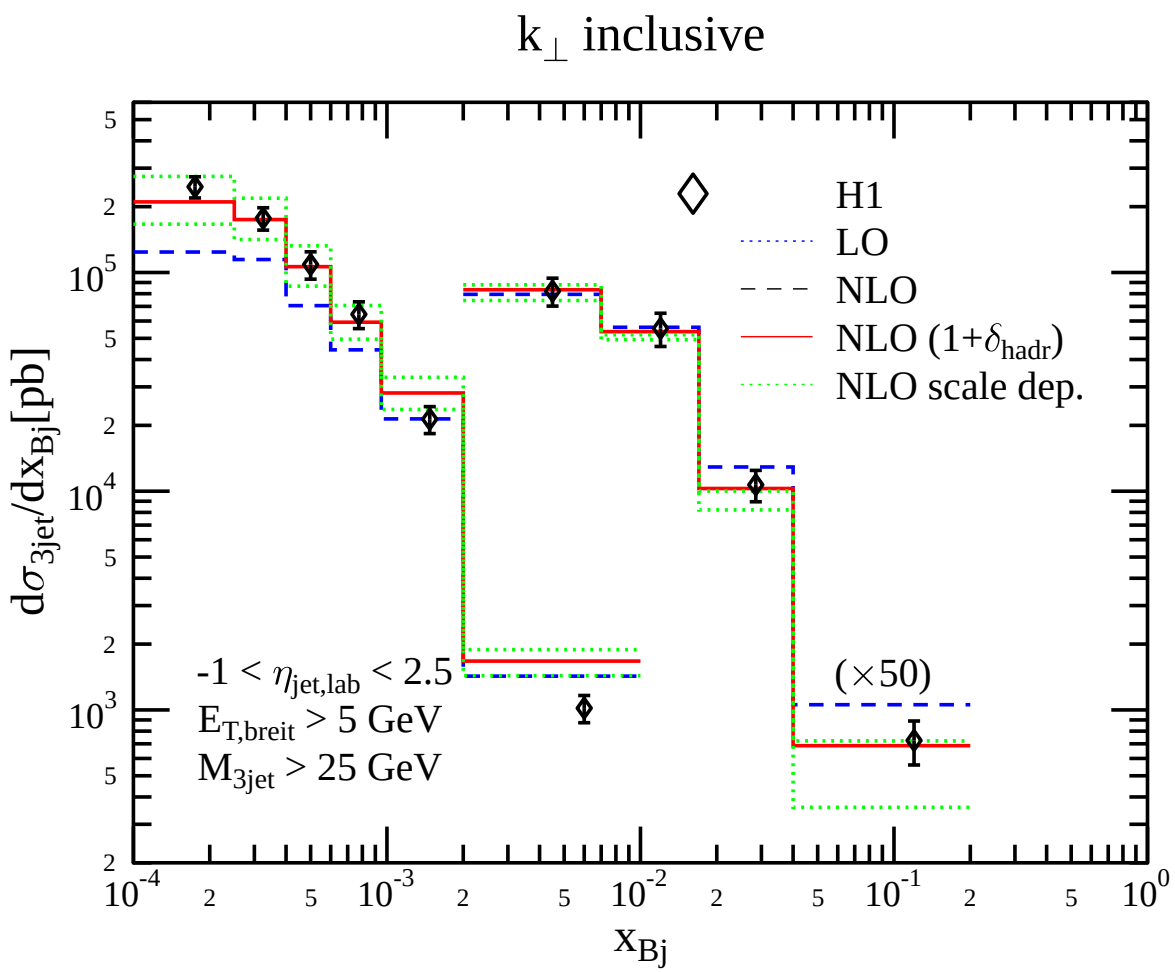
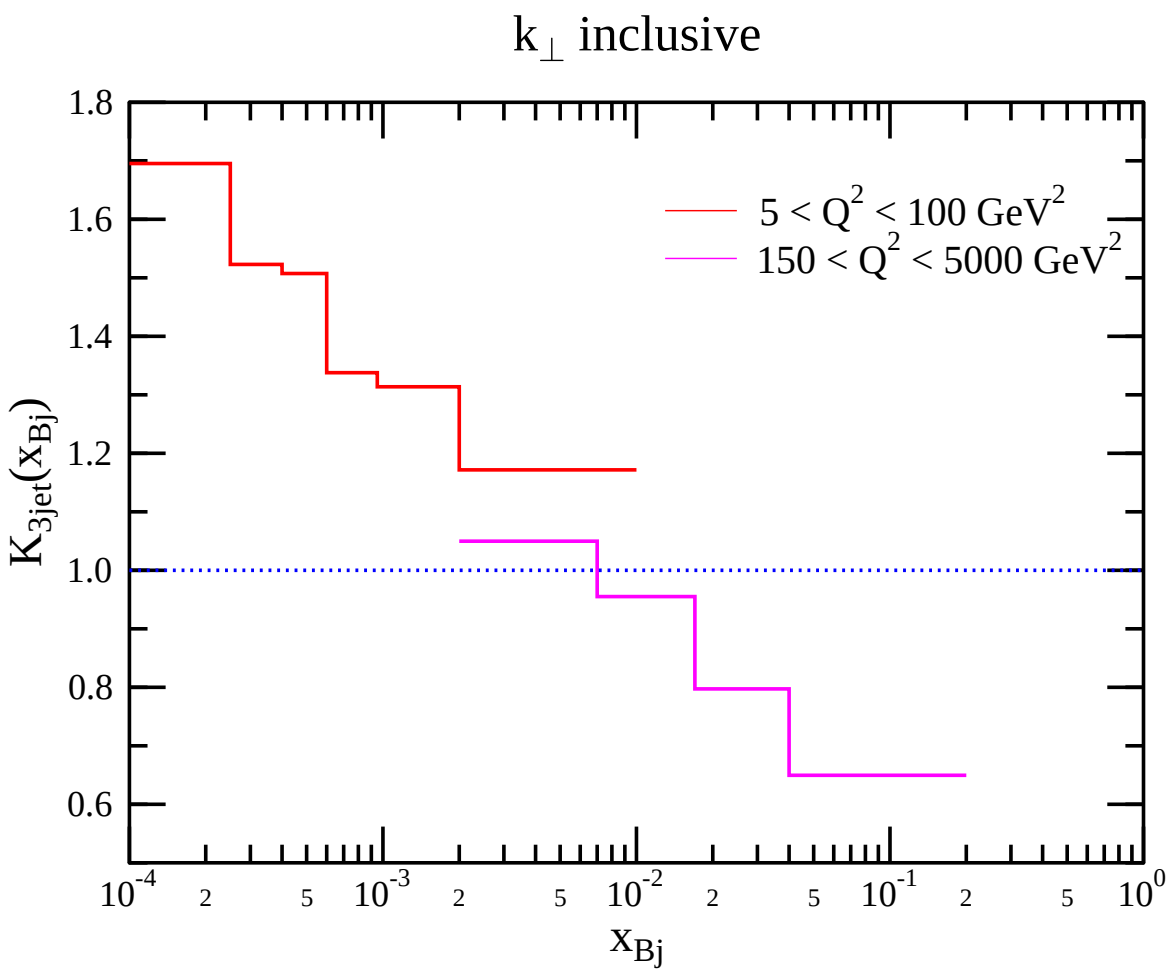
NLO: CTEQ5M1 and two-loop α_s with $\alpha_s(M_{\mathrm{Z}^0}) = 0.118$



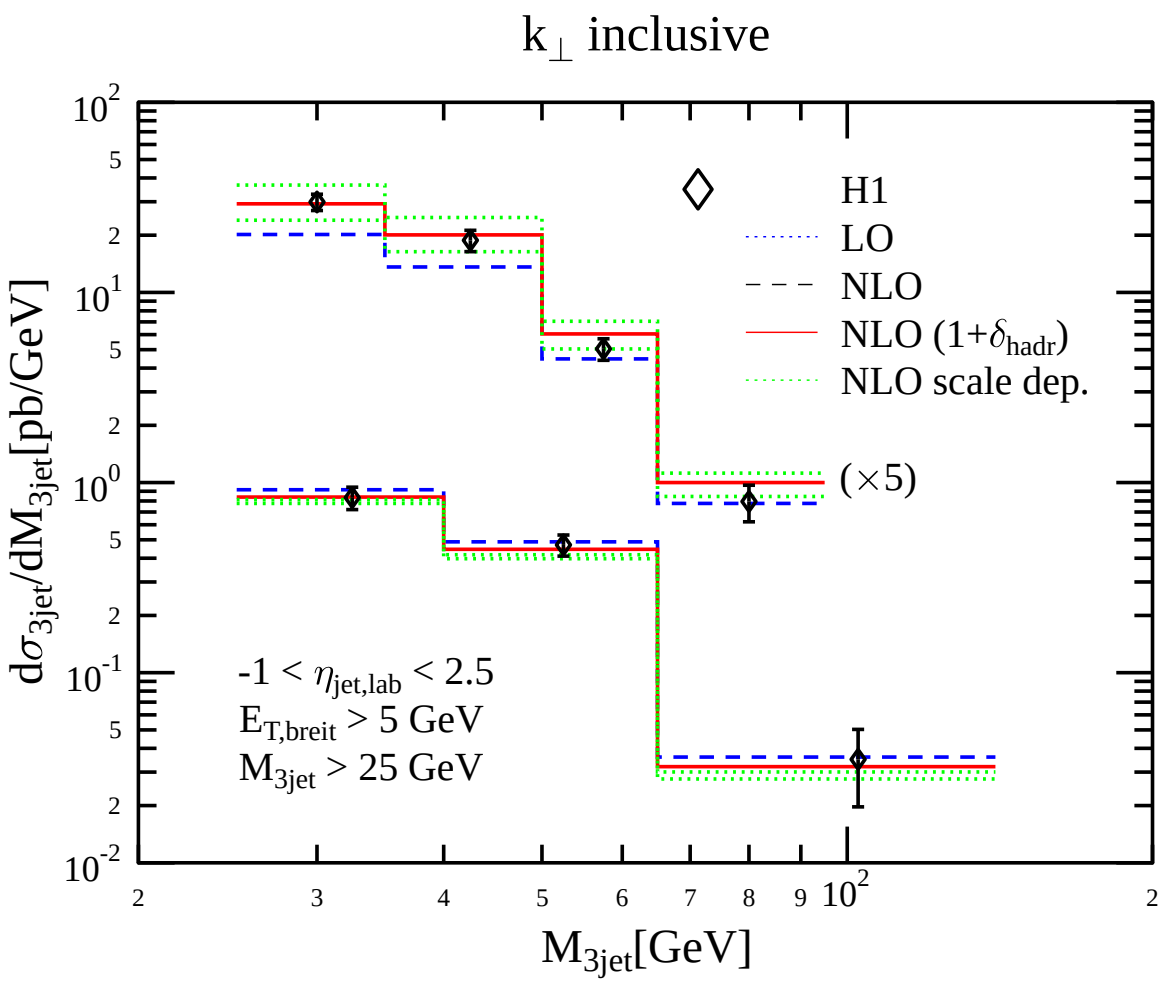
Differential (3+1)-jet cross section as a function of the momentum transfer



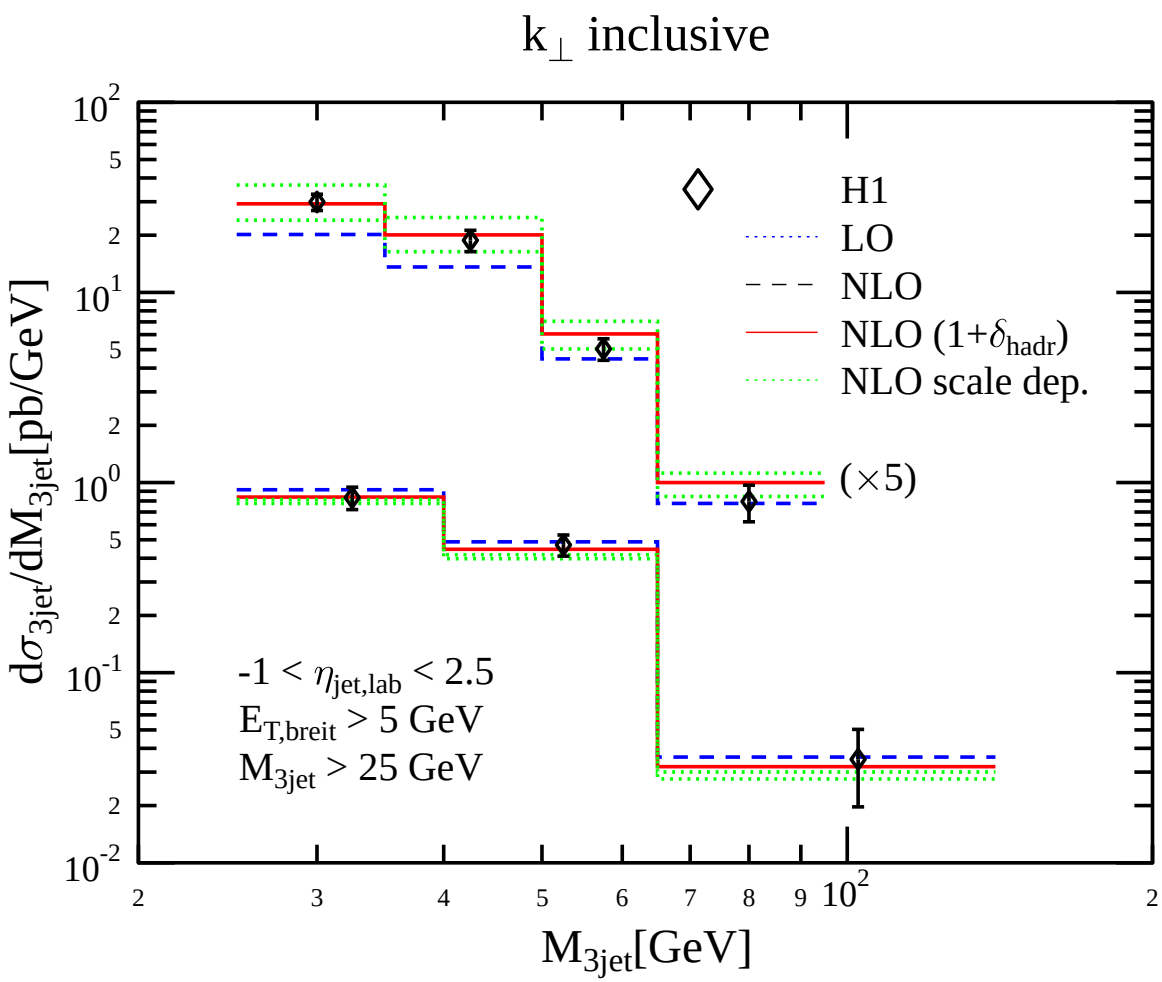
Differential (3+1)-jet cross section as a function of the Bjorken variable



$K_{3\mathrm{jet}} = \sigma^{\mathrm{NLO}}/\sigma^{\mathrm{LO}}$ as a function of the Bjorken variable



Differential (3+1)-jet cross section as a function of the 3-jet invariant mass



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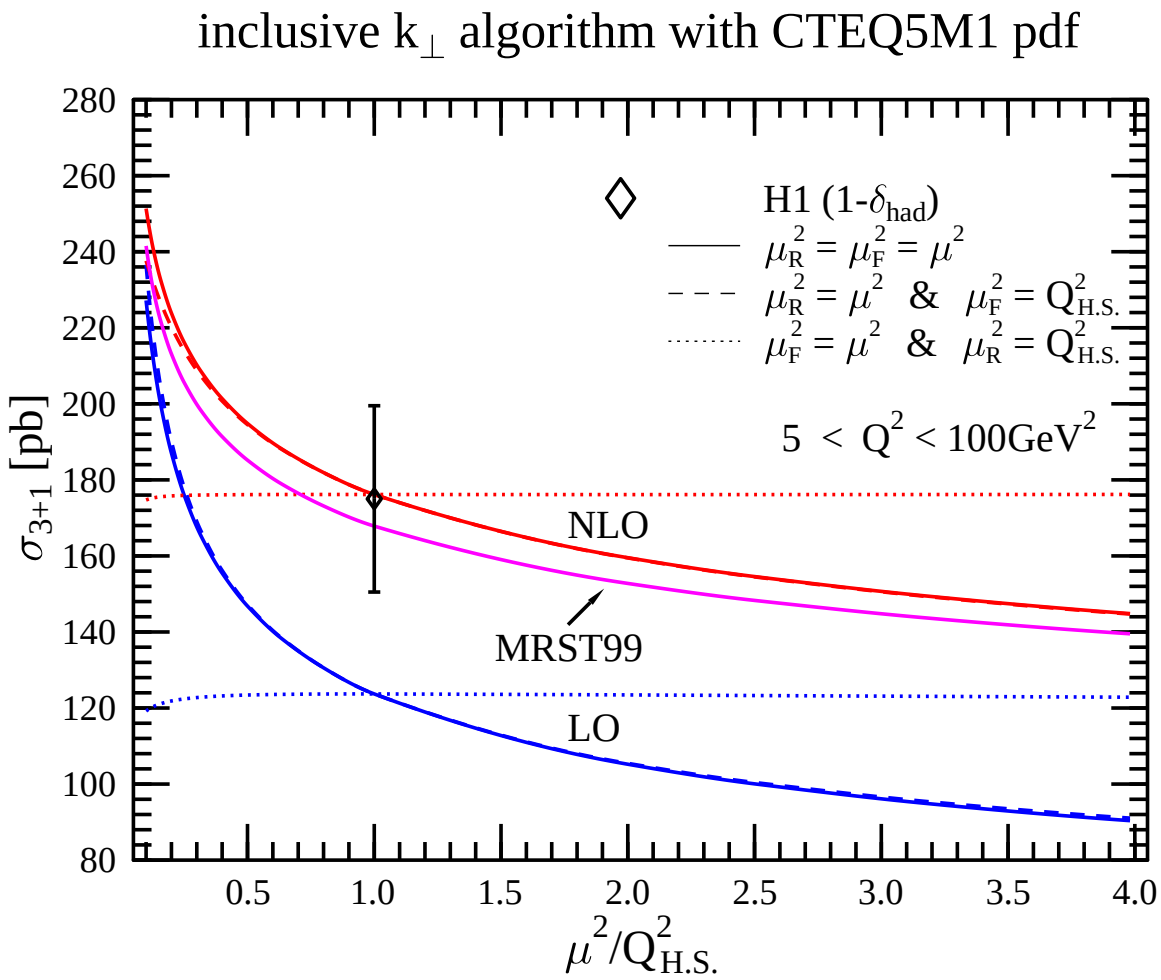
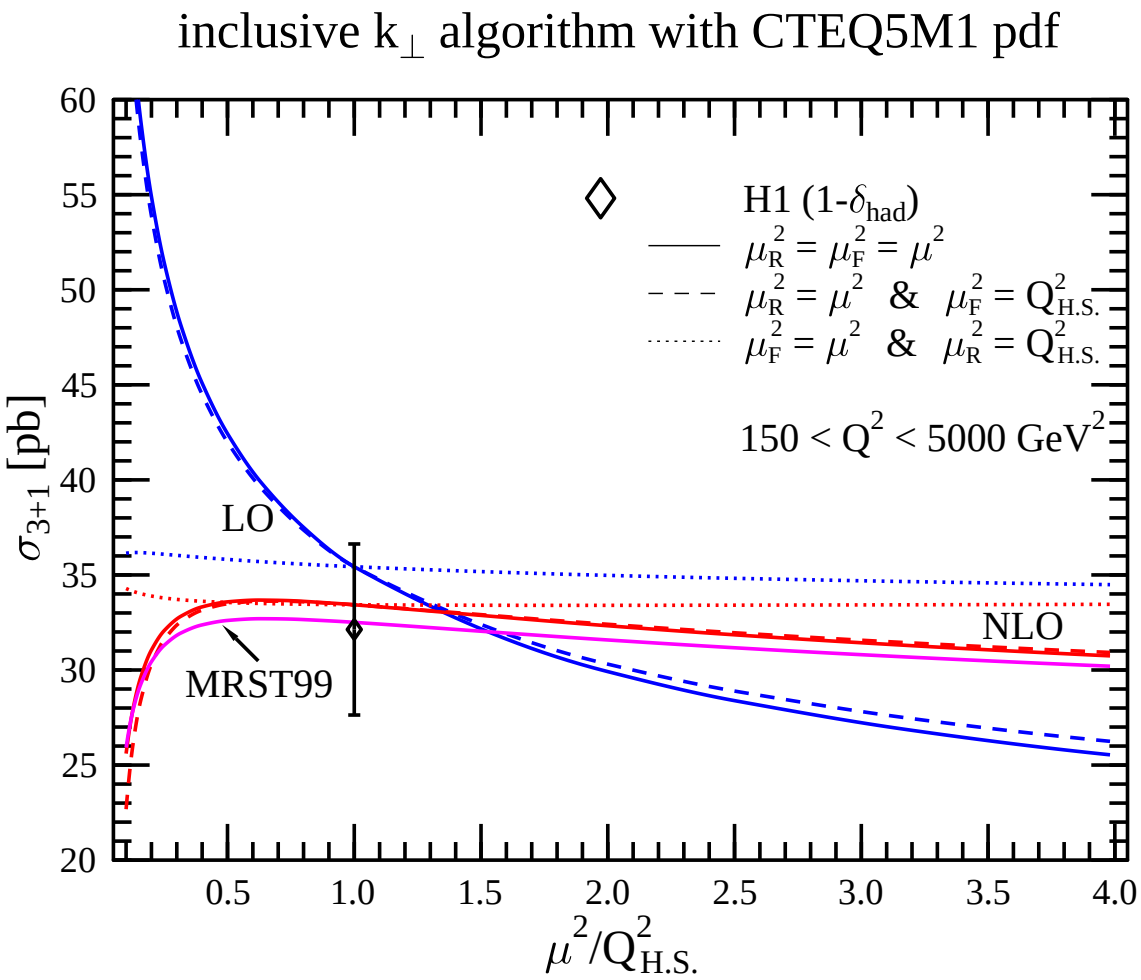


Scale dependence of the inclusive (3+1)-jet cross section at low Q^2



Scale dependence of the inclusive (3+1)-jet cross section at high Q^2

TECHNIQUES FOR NLO CALCULATIONS WITH MASSIVE PARTONS

Extensions of known methods to jet cross sections involving massive partons

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Finite mass M of the QCD partons

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- is a secondary kinematical scale, if the hard scale Q is independent of M (e.g. heavy flavoured jets). The massless limit is IR stable.

ZERO-MASS LIMIT OF QCD JET CROSS SECTIONS

the integral of the real term $\mathrm{d}\sigma^{\mathrm{R}}(M)$ of the NLO cross section in Eq. (6) leads to contributions of the type

$$\int_{m+1} \mathrm{d}\sigma^{\mathrm{R}}(M) \to \int_0^{Q^2} \mathrm{d}\mathbf{q}_\perp^2 \left(\mathbf{q}_\perp^2\right)^{-\varepsilon} \frac{1}{\mathbf{q}_\perp^2 + M^2} \underset{Q \gg M}{\sim} \ln \frac{Q^2}{M^2} + \mathrm{O}(\varepsilon)$$

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$\Rightarrow$ numerical instabilities in

$$\sigma^{\mathrm{NLO}} = \int_{m+1} \left[\left(\mathrm{d}\sigma^{\mathrm{R}}\right)_{\varepsilon=0} - \left(\mathrm{d}\sigma^{\mathrm{A}}\right)_{\varepsilon=0} \right] + \int_m \left[\mathrm{d}\sigma^{\mathrm{V}} + \int_1 \mathrm{d}\sigma^{\mathrm{A}} \right]_{\varepsilon=0}$$

if $M/Q \to 0$, unless

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. . . we choose the auxiliary cross section $\mathrm{d}\sigma^{\mathrm{A}}(M)$ such that

$$\lim_{M\to 0}\int_{m+1}\left[\left(\mathrm{d}\sigma^{\mathrm{R}}(M)\right)_{\varepsilon=0}-\left(\mathrm{d}\sigma^{\mathrm{A}}(M)\right)_{\varepsilon=0}\right]=$$
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$\Rightarrow$ singular terms (ε or $M\to 0$) of 1-loop QCD amplitudes PL **B500**, 149

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  - jet photoproduction (structure functions, two jets)

OUTLOOK

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  - . . . other programs relevant for Higgs and SUSY search, etc