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How Neutrino and Charged Fermion Masses Are Connected Within Minimal Supersymmetric SO(10)

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Massive neutrinos are a generic prediction of SO(10), and supersymmetry is a good partner in unified models. But there is quite a detailed information on neutrino and fermion masses, that the real question is: How/whether it is possible to build a SO(10) supersymmetric model, that correctly incorporates fermion masses. We show that a simple construction of such a model is possible. We concentrate on the two heaviest generations, discuss the predictions of the model, and briefly comment on the pending questions.

theoretical Physics of

... news from $^\vee$ the eighties

Yukawa Couplings at M_{GUT}

$$\mathcal{L} = -\mathbf{16}_i \left[Y_{ij}^{(10)} \underline{\mathbf{10}} + Y_{ij}^{(126)} \underline{\mathbf{126}} \right] \mathbf{16}_j + h.c.$$

Fermions sit in $\mathbf{16}_j$ $(j = 1, 2, 3)$; scalars sit in $\mathbf{10}$ and $\mathbf{126}$.

Georgi, '75
Fritzsch, Minkowski, '75

Where are the useful (=neutral and colorless) scalars?

	SU(2) singlets	SU(2) doublets	SU(2) triplets
10	-	φ_u, φ_d	-
126	S	φ'_u, φ'_d	Δ

Couplings of fermions with these scalars (Yukawa's):

$$\left[\begin{array}{lll} \mathbf{16}_i\ \underline{\mathbf{10}}\ \mathbf{16}_j \propto & \varphi_u\ (u_i^c u_j + \nu_i^c \nu_j) & + \\ & \varphi_d\ (d_i^c d_j + \mathrm{e}_i^c \mathrm{e}_j) & + (i \leftrightarrow j) \\ \mathbf{16}_i\ \underline{\mathbf{126}}\ \mathbf{16}_j \propto & \frac{1}{2} \left(S\ \nu_i^c \nu_j^c + \Delta\ \nu_i \nu_j \right) & + \\ & \varphi'_u\ (u_i^c u_j - 3\ \nu_i^c \nu_j) & + \\ & \varphi'_d\ (d_i^c d_j - 3\ \mathrm{e}_i^c \mathrm{e}_j) & + (i \leftrightarrow j) \end{array}\right]$$

Barbieri et al., '80

... **126**-*plet can make a lot of things!*

Gell-Mann, Ramond, Slansky, '79
Mohapatra, Senjanović, '80
Babu, Mohapatra, '95

Beyond the Great (Supersymmetric) Desert

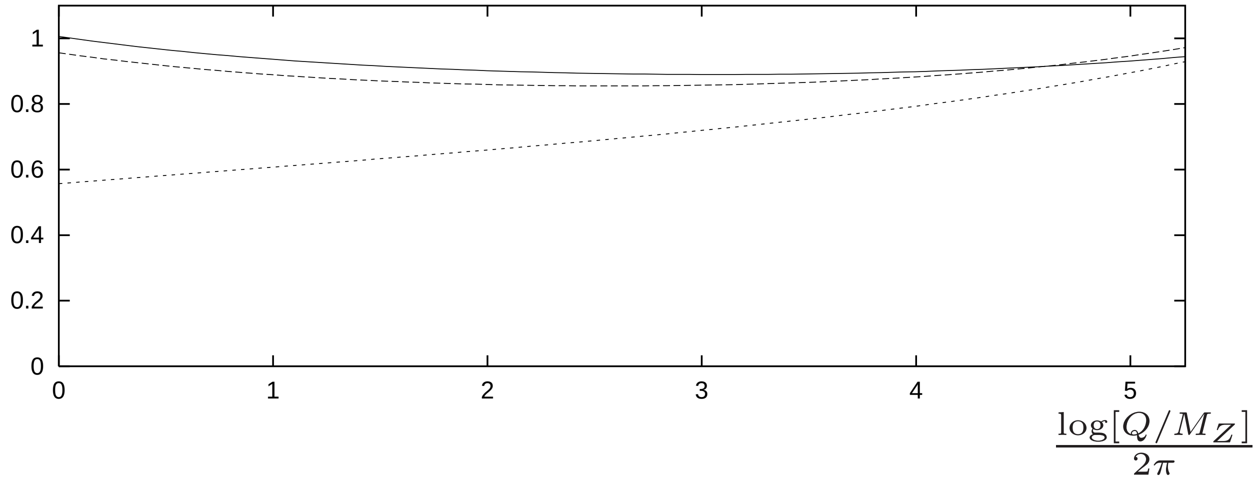
What MSSM (minimal supersymm. SM) wants from SO(10)?

For 3^{rd} family charged fermions mass:

Hypothesis of leading **10**−plet higgs Yukawa, giving

$y_t = y_b = y_\tau$ at M_{GUT} is OK.

Georgi, Nanopoulos, '79



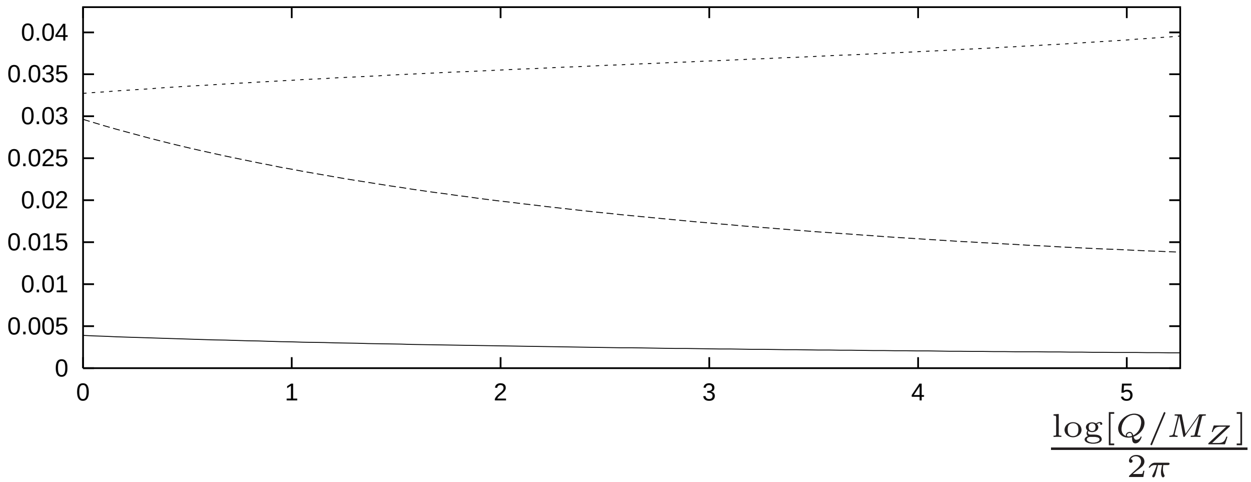
Right-handed ν decoupled (**126** did it); *vevs* ratio $\tan\beta \sim 55$

For 2^{nd} family charged fermion mass:

Hypothesis of leading **126**−plet higgs Yukawa, giving

$y_\mu = -3 \times y_s$ at M_{GUT} is OK (y_c is smaller)

Georgi, Jarlskog, '79



... maybe it is accidental, but suggestive enough to try it.

Determining Model and Parameters

How to match MSSM and SO(10) Yukawa couplings?

MSSM has just 2 light doublets, H_u and H_d.
SO(10) meets MSSM needs if they read $H_u \approx \varphi_u$ and $H_d \approx \varphi_d + \varepsilon\, \varphi_d'$, while orthogonal doublets decouple:

$$\Rightarrow \begin{cases} Y_u \approx Y^{(10)} & \textit{diagonal by definition} \\ Y_d \approx Y^{(10)} + \epsilon\, Y^{(126)} & \\ Y_e \approx Y^{(10)} - 3\, \epsilon\, Y^{(126)} & \end{cases}$$

We know the parameters at M_{GUT}:

y_t, y_b, y_τ	$\Rightarrow$	$Y_{33}^{(10)} \simeq 0.94 \gg \epsilon\, Y_{33}^{(126)}$
y_μ, y_s	$\Rightarrow$	$\epsilon\, Y_{22}^{(126)} \simeq 1.4 \times 10^{-2} > Y_{22}^{(10)}$
y_c	$\Rightarrow$	$Y_{22}^{(10)} \simeq 1.8 \times 10^{-3}$
V_{cb}	$\Rightarrow$	$\epsilon\, Y_{23}^{(126)} \simeq 2.7 \times 10^{-2}$

only $\epsilon\, Y_{33}^{(126)}$ is unknown, though presumably not too far from $\epsilon\, Y_{22}^{(126)}$ or $\epsilon\, Y_{23}^{(126)}$.

... good to know for neutrinos!

Neutrino Properties

What neutrinos want?

Δm^2_{31}	$[1.5, 5] \times 10^{-3}\ \mathrm{eV}^2$	atmospheric neutrinos
Δm^2_{21}	$[2, 50] \times 10^{-5}\ \mathrm{eV}^2$	solar LMA (or $< 2 \times 10^{-7}$ eV^2)
θ_{23}	$[35°, 55°]$	atmospheric neutrinos
θ_{13}	$< 10°$	CHOOZ + atm. (depends on Δm^2_{31})
θ_{12}	$[25°, 43°]$	solar neutrinos (99 % CL)

Super-Kamiokande, MACRO, Soudan,
Homestake, Kamiokande, SAGE, GALLEX/GNO, SNO

A mass matrix with dominant 2-3 block is suggested:

$$\frac{\mathbf{M}_\nu}{\sqrt{\Delta m^2_{31}}} = \frac{1}{2} \times \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{pmatrix} + \mathcal{O}\left(\theta_{13},\ \theta_{23} - \frac{\pi}{4},\ \sqrt{\frac{\Delta m^2_{21}}{\Delta m^2_{31}}}\right)$$

Berezhiani, Rossi, '95
FV, '00

whereas, seesaw suggests $(\mathbf{M}_\nu)_{33}$ dominance in this context:

$$M_D M_R^{-1} M_D = \begin{pmatrix} \epsilon & 0 \\ 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} a & b \\ b & c \end{pmatrix} \cdot \begin{pmatrix} \epsilon & 0 \\ 0 & 1 \end{pmatrix}$$

Babu, Mohapatra, '95

Let us try with triplet, then!

... (we suspect that) what neutrinos want, is
just to be different from the other fermions!

The Triplet Option

At M_Z, the block with 2^{nd} and 3^{rd} generations is:

$$\mathbf{M}_\nu \propto Y^{(126)} \propto \begin{pmatrix} 1 & 1.7 \\ 1.7 & x \end{pmatrix}$$

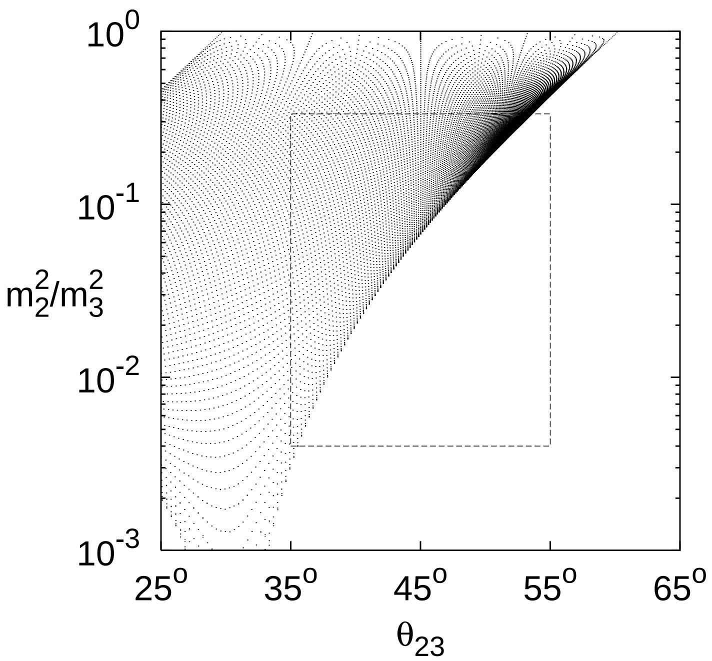
(We have "x", for $Y_{33}^{(126)}$ is unknown, and also because seesaw *might* contribute to 33-entry).

Not incompatible with a dominant block! Thus:

$$\Rightarrow \boxed{\theta_{23} \text{ can be large}}$$

$$\Rightarrow \boxed{\text{we expect a weak mass hierarchy (not } m_3 \gg m_2)}$$

These properties correlate, as can be seen varying $x = \rho \times e^{i\varphi}$:



E.g.: assuming $m_3^2 \simeq \Delta m_{31}^2 = 3 \times 10^{-3}$ eV^2, and $m_2^2 \simeq \Delta m_{21}^2$:

$\star$ If $\theta_{23} = 45°$, then $\Delta m_{21}^2 > 2 \times 10^{-4}$ eV^2;

$\star$ If $\Delta m_{21}^2 = 5 \times 10^{-5}$ eV^2, then $\theta_{23} < 40°$

iff $\sin^2 2\theta_{23} < 0.97\ldots$

Summary and Discussion

$\star$ We discussed an embedding of MSSM into SO(10) with renormalizable Yukawa coupl., where all features of **126**-plet are exploited (=singlet, doublets and triplets vev's all used).

$\star$ Most important step for construction: How the masses of the charged fermions of the 2^{nd} and 3^{rd} generations are explained.

$\star$ Large $\tan\beta$ regime is suggested by 3^{rd} family unification. It is not appealing, but seems still a viable possibility (incidentally, it can accommodate "heavy" higgses, $m_h < 135$ GeV).

$\star$ The triplet mechanism for ν mass generation is at least likely. Correlations $(\mathbf{M}_\nu)_{22} \leftrightarrow m_\mu, m_s$, and $(\mathbf{M}_\nu)_{23} \leftrightarrow V_{cb}$ imply:

$$\theta_{23} \in [\ 35^\circ\ ,\ 55^\circ\ ] \Leftrightarrow \frac{m_2^2}{m_3^2} \in \left[\ \frac{1}{250}\ ,\ \frac{1}{3}\ \right]$$

Solar ν solutions with big hierarchy disfavored (LMA favored). After Δm^2's measurements $\Rightarrow$ upper bound on θ_{23}.

$\star$ Among the pending questions: Masses of 1^{st} family fermions; proton decay rate; baryogenesis through leptogenesis. These important features are strictly linked among them.

... thanks a lot for the attention!
(and wishes to French colleagues)

RGE's for Yukawa's at 1 loop

Relevant supersymmetric RGE's are:

$$
\begin{cases}
\alpha_t' = \alpha_t[6\alpha_t + \alpha_b - 16/3\alpha_3 - 3\alpha_2 - 13/9\alpha_1] \\
\alpha_b' = \alpha_b[6\alpha_b + \alpha_t + \alpha_\tau - 16/3\alpha_3 - 3\alpha_2 - 7/9\alpha_1] \\
\alpha_\tau' = \alpha_\tau[4\alpha_\tau + 3\alpha_b - 3\alpha_2 - 3\alpha_1] \\
\\
\alpha_c' = \alpha_c[3\alpha_t - 16/3\alpha_3 - 3\alpha_2 - 13/9\alpha_1] \\
\alpha_s' = \alpha_s[3\alpha_b - 16/3\alpha_3 - 3\alpha_2 - 7/9\alpha_1] \\
\alpha_\mu' = \alpha_\mu[\alpha_\tau + 3\alpha_b - 3\alpha_2 - 3\alpha_1] \\
\\
A' = -A[\alpha_t + \alpha_b]/2 \\
\lambda' = 0 \\
\rho' = 0 \\
\eta' = 0 \\
\\
M_{ij}' = M_{ij}[\alpha_\tau(k_i + k_j)/2 + 3\alpha_t - 3\alpha_2 - \alpha_1]
\end{cases}
$$

Prime denotes derivative with respect to
$T = \log(Q/M_Z)/(2\pi)$.

$\alpha_x = y_x^2/4\pi$ for $x = t, b, \tau, c, s, \mu$, analogous with gauge α_i's.
A, λ, η, ρ =Wolfenstein parameterization of quark mixings.
M_{ij} =entries of ν_L mass matrix; $k_3 = 1$, and $k_2 = 0$.

α_1 is normalized in standard model fashion–not SU(5)'s.

Some Stray Numbers

$$M_{GUT} = 2 \times 10^{16}\ \text{GeV}$$

$$\alpha_s = 0.118$$

$$\tan\beta = 55.4$$

	y_t	y_b	y_τ	$y_c \cdot 10^3$	$y_s \cdot 10^2$	$y_\mu \cdot 10^2$
M_Z	1.0	.96	.56	3.9	3.0	3.3
M_{GUT}	.94	.96	.92	1.8	1.5	4.0

	$V_{cb} \cdot 10^2$	M_{33}	M_{23}	M_{22}
M_Z	4.0	1	1	1
M_{GUT}	2.8	1.83	1.64	1.48