Radiative Lepton Decays
and
the Substructure of Leptons

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- BNL measurement of the anomalous magnetic moment of the muon μ^+

$$\Delta a_\mu = a_\mu(exp) - a_\mu(SM) = (4.3 \pm 1.6) \times 10^{-9}.$$

- This is only a 2.6 σ excess and clearly some work (theoretical/experimental) is needed to confirm this deviation.
- If this result is confirmed, it can be interpreted as the first signal towards an internal structure of the leptons.
- The contribution to $g-2$ of the muon can be described by adding an effective term $\mathcal{L}^{eff}$ to the Lagrangian of the SM:

$$\mathcal{L}^{eff} = \frac{e}{2\Lambda}\bar{\mu}\left(A + B\gamma_5\right)\sigma_{\mu\nu}\mu F^{\mu\nu},$$

Λ is the compositeness scale. A and B are determined by dynamical details of the underlying composite structure. $A \approx 1$ (compare with QCD), B can be constrained by EDM measurements.

- We get

$$\Delta a_\mu = \left(\frac{m_\mu}{\Lambda}\right),$$

and thus $\Lambda \approx 2.5 \times 10^7$ GeV

- But, the $g-2$ term has the same chiral structure as the lepton mass term $\to$ the same mechanism which leads to the small lepton masses ($m_\mu \ll \Lambda$), e.g. a chiral symmetry, leads to a corresponding suppression of $g-2$:

$$\mathcal{L}^{eff} = \frac{e}{2\Lambda}\frac{m_\mu}{\Lambda}\bar{\mu}\left(A + B\gamma_5\right)\sigma_{\mu\nu}\mu F^{\mu\nu} \times\left(1-\frac{4\alpha}{\pi}\ln\frac{\Lambda}{m_\mu}\right).$$

- The contribution of the muon substructure to $g-2$ is in this case given by

$$\Delta a_\mu = \left(\frac{m_\mu}{\Lambda}\right)^2 f^{\text{QED}}$$

with

$$f^{\text{QED}} = \left(1-\frac{4\alpha}{\pi}\ln\frac{\Lambda}{m_\mu}\right).$$

One obtains: $\Lambda \approx 1.54$ TeV, i.e. Λ is much smaller due to the chiral symmetry argument. The 95% confidence level range for Λ is 1.16 TeV $< \Lambda < 3.04$ TeV.

- If the leptons have a composite structure, the question arises whether flavor-changing transitions, e.g. the decays $\mu \to e\gamma$ or $\tau \to \mu\gamma$ arise.

- First consider the limit $m_e = m_\mu = 0$ and $m_\tau \neq 0 \to$ the mass matrix for the charged leptons has the structure $m_{l^-} = m_\tau \text{diag}(0,0,1)$ and exhibits a democratic symmetry. $\exists$ a chiral symmetry $SU(2)_L \otimes SU(2)_R$ acting on the first two lepton flavors.

- The magnetic moment term induced by compositeness, being of a similar chiral nature as the mass term, must respect this symmetry:

$$\mathcal{L}^{eff} = \frac{e}{2\Lambda}\frac{m_\tau}{\Lambda}\bar{\psi}\tilde{M}\,(A + B\gamma_5)\,\sigma_{\mu\nu}\psi F^{\mu\nu} f^{\text{QED}},$$

  $\psi = (e, \mu, \tau)$ and $\tilde{M} = \text{diag}(0, 0, 1)$.

- When the chiral symmetry is broken $\rightarrow$ mass matrix receives non-zero entries, after diagonalization it takes the form $M = \text{diag}(m_e, m_\mu, m_\tau)$.

- If after symmetry breaking the mass matrix M and the magnetic moment matrix $\tilde{M}$ were identical, the same diagonalization procedure which leads to a diagonalized mass matrix would lead to a diagonalized magnetic moment matrix.

- However there is no reason why $\tilde{M}$ and M should be proportional to each other after symmetry breaking:
  $\exists$ flavor-non-diagonal terms e.g. terms proportional to
  $\bar{e}\ \sigma_{\mu\nu}\,(A + B\gamma_5)\,\mu$.

- These flavor-non-diagonal term must obey the constraints imposed by the chiral symmetry:
  - they disappear once the masses of the light leptons involved are turned off. e.g., $e - \mu$ transition must vanish for $m_e \to 0$
  - the flavor changing terms arise due to a mismatch between the mass density and the g-2 operators due to the internal substructure:
    in the limit $\Lambda \to \infty$, the effects should not be present.
- The simplest Ansatz for the transition terms between the leptons flavors i and j is $const.\sqrt{m_i m_j}/\Lambda$. It obeys the constraints mentioned above:
  - it vanishes once the mass of one of the leptons is turned off
  - it is symmetric between i and j and it vanishes for $\Lambda \to \infty$.
- In this case the magnetic moment operator has the general form:

$$
\begin{aligned}
\mathcal{L}^{eff} \quad = \quad & \frac{e}{2\Lambda}\frac{m_\tau}{\Lambda}\bar{\psi}\begin{pmatrix} \frac{m_e}{m_\tau} & C_{e\mu}\frac{\sqrt{m_e m_\mu}}{\Lambda} & C_{e\tau}\frac{\sqrt{m_e m_\tau}}{\Lambda} \\ C_{e\mu}\frac{\sqrt{m_e m_\mu}}{\Lambda} & \frac{m_\mu}{m_\tau} & C_{\mu\tau}\frac{\sqrt{m_\mu m_\tau}}{\Lambda} \\ C_{e\tau}\frac{\sqrt{m_e m_\tau}}{\Lambda} & C_{\mu\tau}\frac{\sqrt{m_\mu m_\tau}}{\Lambda} & 1 \end{pmatrix} \\
& \cdot\psi\,(A + B\gamma_5)\,\sigma_{\mu\nu}F^{\mu\nu} f^{\mathrm{QED}}.
\end{aligned}
$$

Here C_{ij} are constants of the order one. In general one may introduce two different matrices (with different constants C_{ij}) both for the 1-term and for the γ_5-term, but we shall limit ourselves to the simpler structure given above.

- Based on the flavor-changing transition terms, we can calculate the branching ratios for the decays $\mu \to e\gamma$, $\tau \to \mu\gamma$ and $\tau \to e\gamma$:

$$
\begin{aligned}
\Gamma(\mu \to e\gamma) &= e^2 \frac{m_\mu}{8\pi} \left(\frac{\sqrt{m_\mu m_e}}{\Lambda} \right)^2 \left(\frac{m_\mu}{\Lambda} \right)^2 \left(\frac{m_\tau}{\Lambda} \right)^2 \\
&\quad \times \left(|A|^2 + |B|^2 \right) \left(f^{\text{QED}} \right)^2, \\
\Gamma(\tau \to \mu\gamma) &= e^2 \frac{m_\tau}{8\pi} \left(\frac{\sqrt{m_\tau m_\mu}}{\Lambda} \right)^2 \left(\frac{m_\tau}{\Lambda} \right)^2 \left(\frac{m_\tau}{\Lambda} \right)^2 \\
&\quad \times \left(|A|^2 + |B|^2 \right) \left(f^{\text{QED}} \right)^2, \\
\Gamma(\tau \to e\gamma) &= e^2 \frac{m_\tau}{8\pi} \left(\frac{\sqrt{m_\tau m_e}}{\Lambda} \right)^2 \left(\frac{m_\tau}{\Lambda} \right)^2 \left(\frac{m_\tau}{\Lambda} \right)^2 \\
&\quad \times \left(|A|^2 + |B|^2 \right) \left(f^{\text{QED}} \right)^2.
\end{aligned}
$$

- $|B|$ can be constrained using the limits for the electron electric dipole moment (EDM). We get

$$
d_e = \frac{e}{2\Lambda} \frac{m_e}{\Lambda} |B| \left(f^{\text{QED}} \right) = 1.9 \times 10^{-24} |B| \text{ e-cm},
$$

which has to be compared to the experimental limit $d_e{}^{\text{exp}} < (0.18 \pm 0.12 \pm 0.10) \times 10^{-26} e -$ cm, thus $|B| \ll |A|$: we set $|B| = 0$.

- The corresponding branching ratios are:

$$
\begin{aligned}
\text{Br}(\mu \to e\gamma) &\approx 1.5 \times 10^{-10}, \\
\text{Br}(\tau \to \mu\gamma) &\approx 3.5 \times 10^{-10}, \\
\text{Br}(\tau \to e\gamma) &\approx 1.7 \times 10^{-12},
\end{aligned}
$$

using the central value of Δa_μ to evaluate Λ.

- Using the 95% confidence level range for Λ, we get

$$\begin{aligned} 8.3\times10^{-10} &> \mathrm{Br}(\mu\to e\gamma) &> 2.5\times10^{-12}, \\ 1.9\times10^{-9} &> \mathrm{Br}(\tau\to \mu\gamma) &> 5.8\times10^{-12}, \\ 9.3\times10^{-12} &> \mathrm{Br}(\tau\to e\gamma) &> 2.8\times10^{-14}. \end{aligned}$$

- The upper part of the range for the $\mu \to e\gamma$ decay is excluded by the present experimental limit: $\mathrm{Br}(\mu \to e\gamma) < 1.2\times10^{-11}$.

- Our estimates of the branching ratio should be viewed as order of magnitude estimates. In general we can say that the branching ratio for the $\mu \to e\gamma$ decay should lie between 10^{-13} and the present limit.

- The decay $\tau \to \mu\gamma$ processes at a level which cannot be observed, at least not in the foreseeable future. The decay $\tau \to e\gamma$ is, as expected, much suppressed compared to $\tau \to \mu\gamma$ decay and cannot be seen experimentally.

- The experiment now under way at the PSI should be able to detect this decay. If it is found, it would be an important milestone towards a deeper understanding of the internal structure of the leptons and quarks.