Bulk Observers in Non-Factorizable Geometries

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Based on:

B. Grinstein and D.R. Nolte, hep-th/0106163

B. Grinstein, D.R. Nolte, W. Skiba, Phys.Rev.D63:105016(2001)

Phys.Rev.D63:105005,2001

Dedicated to the Memory of Detlef Nolte

Disclaimer

In this talk I won't propose any new physics. I will try to give a physically intuitive picture of known physics.

Consider a world with non-factorizable geometry. For defitiness, a RS-I model:

$$ds^2 = G_{MN} dx^M dx^N = a^2(y) \eta_{\mu\nu} dx^\mu dx^\nu - dy^2$$

This solves Einstein's equations provided

$$a(y) = e^{-k|y|}$$

and

$$V_{hid} = -V_{vis} = 24 M^3 k \quad \Lambda = -24 M^3 k^2$$

(ie, fine tune all cosmological constants).

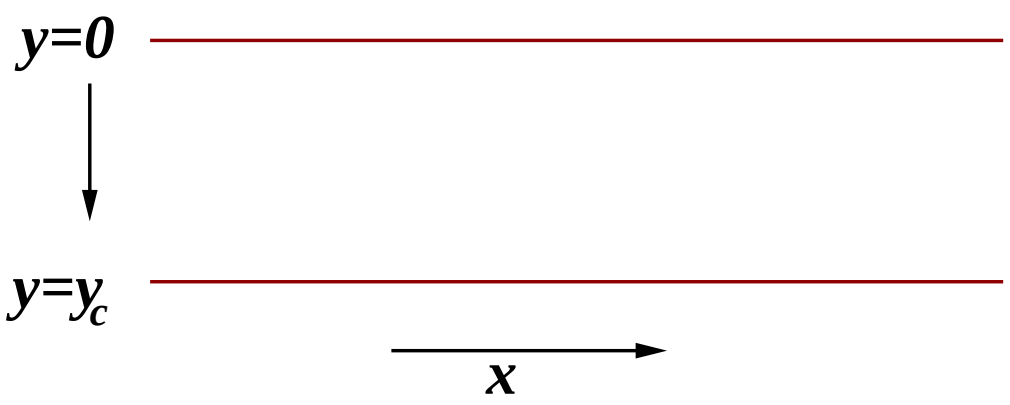
• y-direction ("transverse") microscopic: unresolved by macroscopic observers.

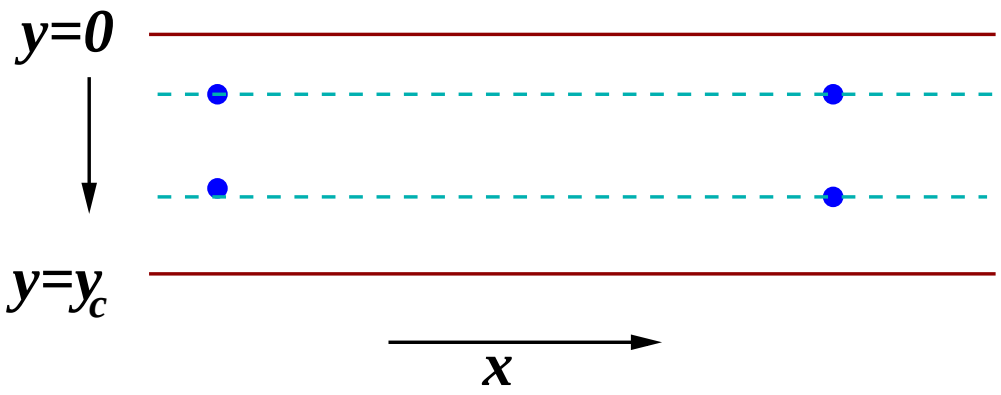
Questions

- Macrocopic observer ignores y? Doesn't distance between $(0, y_1)$ and $(\Delta x, y_2)$ depend on y_1 and y_2 (figure).
- So, what is $\phi(x)$?
- What is mass?

$$\langle \phi(x)\phi(0)\rangle_E \sim e^{-m|x|}$$

- How does a macroscopic object hang together, if extended over y?





Brane and Bulk Observers

- Observer defined by their measuring apparatus
- Must differentiate between apparatus made of stuff restricted to a brane, “Brane Observer”, and made from stuff that lives in bulk, “Bulk Observer”
- Each measures distance by extending a rope between two points
- They can measure time by bouncing light between mirrors at an already determined fixed distance appart

Geodesics

Geodesic equation for $0 < y < y_c$
(obtain $-y_c < y < 0$ formally by $k \to -k$):

$$\begin{aligned} \frac{d^2 x^\mu}{d\tau^2} - 2k \frac{dy}{d\tau} \frac{dx^\mu}{d\tau} &= 0 \\ \frac{d^2 y}{d\tau^2} - k e^{-2ky} \eta_{\mu\nu} \frac{dx^\mu}{d\tau} \frac{dx^\nu}{d\tau} &= 0 \end{aligned}$$

Solution for $\eta_{\mu\nu} \frac{dx^\mu}{d\tau} \frac{dx^\nu}{d\tau} > 0$: time-like geodesics are

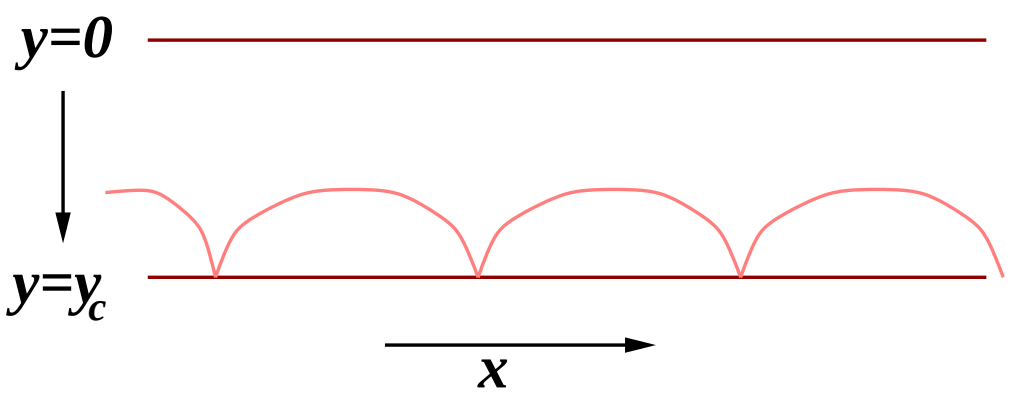
$$\begin{aligned} x^\mu &= x_0^\mu + \frac{v_0^\mu}{k v_0^2} \tan(k\tau) \\ y &= -\frac{1}{2k} \ln(v_0^2 \cos^2(k\tau)) \end{aligned}$$

- $v_0^2 \equiv \eta_{\mu\nu} v_0^\mu v_0^\nu$
- v_0^μ and x_0^μ constants of integration
- Freedom $\tau \to \tau - \tau_0$, chosen $\tau_0 = 0$ so that $dy/d\tau = 0$ at $\tau = 0$

- $dy/d\tau|_{y_c}$ continuous $\Rightarrow$ if we insist in describing the motion on the interval $[0, y_c]$, the particle bounces off the brane at $y = y_c$ reversing its velocity $dy/d\tau$
- $y(\tau)$ periodic, with period $\frac{2}{k}\cos^{-1}(e^{-ky_c}/\sqrt{v_0^2})$
- In terms of the coordinate time $t = x^0$

$$
\begin{aligned}
x^i &= x_0^i + \frac{v_0^i}{v_0^0}(t - t_0) \\
y &= \frac{1}{2k}\ln\left(\frac{1}{v_0^2} + \frac{v_0^2}{v_0^{02}}k^2(t - t_0)^2\right)
\end{aligned}
$$

- Uniform motion in $x \Rightarrow$ Bulk stuff OK



Distances

Measure distance $\Leftarrow$ choose frame

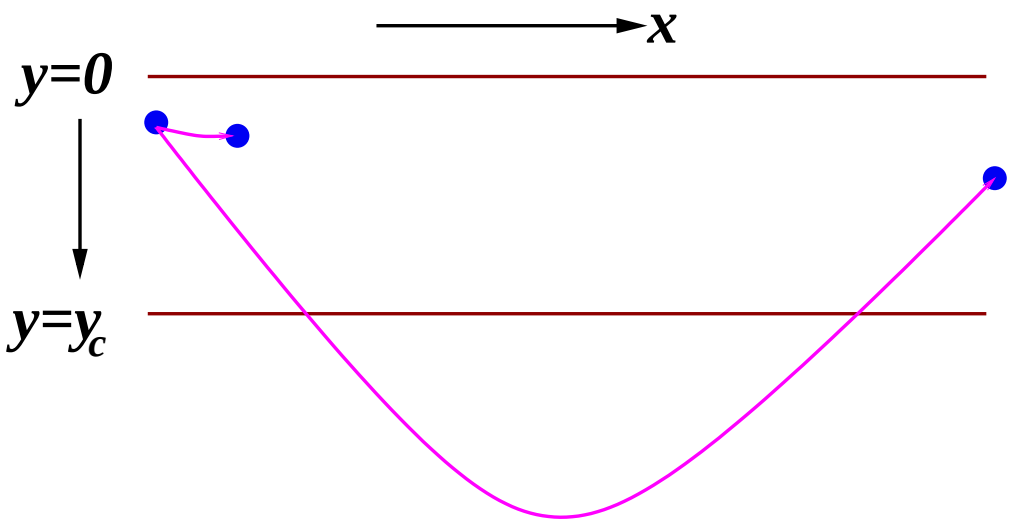
$\Rightarrow$ Geodesics on fixed-t hypersurface

$$\begin{aligned} \frac{d^2x^i}{d\tau^2} - 2k\frac{dy}{d\tau}\frac{dx^i}{d\tau} &= 0 \\ \frac{d^2y}{d\tau^2} + ke^{-2ky}\delta_{ij}\frac{dx^i}{d\tau}\frac{dx^j}{d\tau} &= 0 \end{aligned}$$

Solutions

$$\begin{aligned} x^i(\tau) &= x_0^i + \frac{\beta^i}{k^2\beta^2}\tanh[\omega(\tau-\tau_0)] \\ y(\tau) &= -\frac{1}{k}\ln(k\beta\cosh[\omega(\tau-\tau_0)]) \end{aligned}$$

- $\beta \equiv \sqrt{\delta_{ij}\beta^i\beta^j}$
- τ is an affine parameter, $\tau \in [0,1]$
- ω is physical length ℓ in units of k along the geodesic, $\omega = k\ell$



Parameters ω, β, τ_0 determined by

$$
\begin{aligned}
k\beta &= \frac{|\tanh[\omega(1-\tau_0)] + \tanh[\omega\tau_0]|}{k|\Delta x|} \\
\omega(1-\tau_0) &= \text{arcsinh}\left(\frac{d_1^2 - 1 + d_1^2/d_0^2}{2d_1}\right) \\
\omega\tau_0 &= \text{arcsinh}\left(\frac{d_0^2 - 1 + d_0^2/d_1^2}{2d_0}\right)
\end{aligned}
$$

where

$$|\Delta x|^2 \equiv \delta_{ij}(x^i(1) - x^i(0))(x^j(1) - x^j(0))$$

$$
\begin{aligned}
d_0 &= k|\Delta x|e^{-ky(0)} \\
d_1 &= k|\Delta x|e^{-ky(1)}
\end{aligned}
$$

(lengths in units of k between $x^i(0)$ and $x^i(1)$ along curves of fixed y)

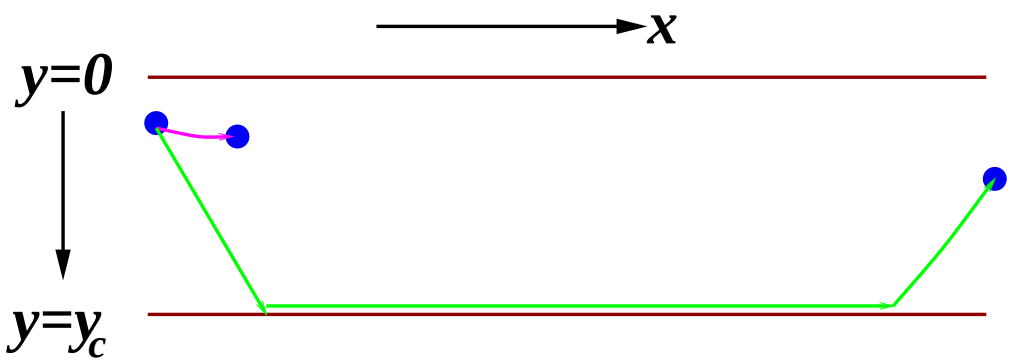
Interpretation clearest in picture above

Analysis (in case of questions)

Consider case $y(0) \leq y(1)$, described by $\tau_0 \geq 1/2$.
If $\tau_0 \geq 1$ then $y(\tau)$ increases monotonically from $y(0)$ to $y(1)$
If $1/2 \leq \tau_0 < 1$ the solution $y(\tau)$ increases monotonically from $y(0)$ beyond $y(1)$ (at $\tau = 2\tau_0 - 1$) to a maximum at $y(\tau_0)$ and decreases monotonically back to $y(1)$
Transition at $\tau_0 = 1$, critical x separation given by

$$k|\Delta x|_{\mathrm{crit}} e^{-ky(1)} = \sqrt{1 - e^{-2k(y(1)-y(0))}}$$

(For $|\Delta x| < |\Delta x|_{\mathrm{crit}}$ the solution has $\tau_0 > 1$)



At large Δx the shortest path is actually along a $\tau_0 = 1$ geodesic from $y(0)$ to y_c, then along the brane $y = y_c$ and finally back from y_c to $y(1)$ along a second ($\tau_0 = 0$) geodesic. The shortest path between two points has length

$$\ell_{\rm phys} = |\Delta x| e^{-k y_c} + (2y_c - y(0) - y(1) + \ln 4 - 2) + \mathcal{O}(e^{-k(y_c - \min(y(0), y(1))})$$

Answers

- A bulk observer made of many particles: they all move at uniform velocity in x direction.
- Bulk observer's meter stick does not depend on the y-coordinates of end points (error smaller than transverse width)
- Distance measured by bulk observer is same as for negative tension brane observer ($y = y_c$ brane)
- Similar conclusions for time measurements, since motion is uniform in $x^i(t)$, regardless y
- Effective action, $\phi(x)$, masses: next slide

Bulk Scalar Action:

$$S = \frac{1}{2}\int d^4x dy \sqrt{G}\left(G^{AB}\partial_A\Phi\partial_B\Phi - M^2\Phi^2\right)$$

Ozeki and Shimoyama (after Goldberger and Wise) bulk scalar effective action: use decomposition

$$\Phi(x,y) = \sum_n \psi_n(x)\xi_n(y)$$

normalized so that the effective action is written in terms of the induced metric g_{ind} according to

$$\int_{-y_c}^{y_c} dy\, e^{-2k|y|}\xi_n(y)\xi_m(y) = \begin{cases} e^{-2k|y_c|}\delta_{nm} & \text{observer at } y_c \\ \delta_{nm} & \text{observer at } y_0 \end{cases}$$

and which satisfy

$$-\frac{d}{dy}\left(e^{-4k|y|}\frac{d\xi_n}{dy}\right) + M^2 e^{-4k|y|}\xi_n = m_n^2 e^{-2k|y|}\xi_n$$

giving

$$S = \frac{1}{2}\sum_n \int d^4x \sqrt{g_{ind}}\left[g_{ind}^{\mu\nu}\partial_\mu\psi_n\partial_\nu\psi_n - m_n^2\psi_n^2\right]$$

Is this right? And, What about Bulk Observers?

Green Functions

$$\begin{aligned}\Delta(x,y;x',y') &= -\sum_n \int \frac{d^4q}{(2\pi)^4}\frac{e^{iq\cdot(x-x')}}{q^2-m_n^2}R_n(y,y') \\ &= \sum_n \Delta^{(4)}(x-x';m_n)R_n(y,y')\end{aligned}$$

$R_n(y,y') = r_n(y)r_n(y') \Rightarrow \Delta(x,y;x',y')$ as in 4D with source $j(x)$ coupling to $\sum_n r_n(y)\psi_n(x)$

The spectrum, m_n:

$$\tilde{N}_\nu(m_n z_1)\tilde{J}_\nu(m_n z_2) - \tilde{J}_\nu(m_n z_1)\tilde{N}_\nu(m_n z_2) = 0$$

where $z_{1,2} = e^{ky}/k|_{y=0,y_c}$, $\nu = \sqrt{4+m^2/k^2}$, and

$$\tilde{Z}_\nu(z) = (1-\frac{\nu}{2})Z_\nu(z) + \frac{z}{2}Z_{\nu-1}(z)$$

same as Goldberger-Wise

- $m_n \sim a(y_c)k$
- $y = y_c$-brane observer, $m_n^{\rm phys} = m_n/a(y_c) \sim k$
- $y = 0$-brane observer, $m_n^{\rm phys} = m_n \sim a(y_c)k$
- Bulk observer just as $y = y_c$-brane observer, $m_n^{\rm phys} \sim k$
- Effective action for bulk observer just as for $y = y_c$-brane observer and $\phi(x)$ with x as in $y = y_c$ brane

Brane Fields

- While at it, consider

$$S = \frac{1}{2}\int d^4x dy \sqrt{G}\delta(\sqrt{G_{55}}(y-y_c)) \times \left(G^{AB}\partial_A\Phi\partial_B\Phi - M^2\Phi^2\right)$$

  plus same at $y = 0$

- Rescaling field $\Phi \to \Phi/a(y_c)$ gives

$$S = \frac{1}{2}\int d^4x \left(\eta^{\mu\nu}\partial_\mu\Phi\partial_\nu\Phi - (a(y_c)M)^2\Phi^2\right)$$

- However, $M^{\rm phys} = M$
  (not $= a(y_c)M = e^{-ky_c}M$)

- And at $y = 0$, $M^{\rm phys} = M$

- Hierarchy comes from $G_N \sim a(y_c)$

- Irrelevant? Are all ratios the same? Don't think so: consider glueball mass. Both methods give $\Lambda \sim M\exp(-8\pi^2/b_0)$ in both branes.

Other Applications

- Model with brane stabilization:

$$ds^2 = n^2(t,y)dt^2 - a^2(t,y)d\vec{x}^2 - b^2(t,y)dy^2$$

with action

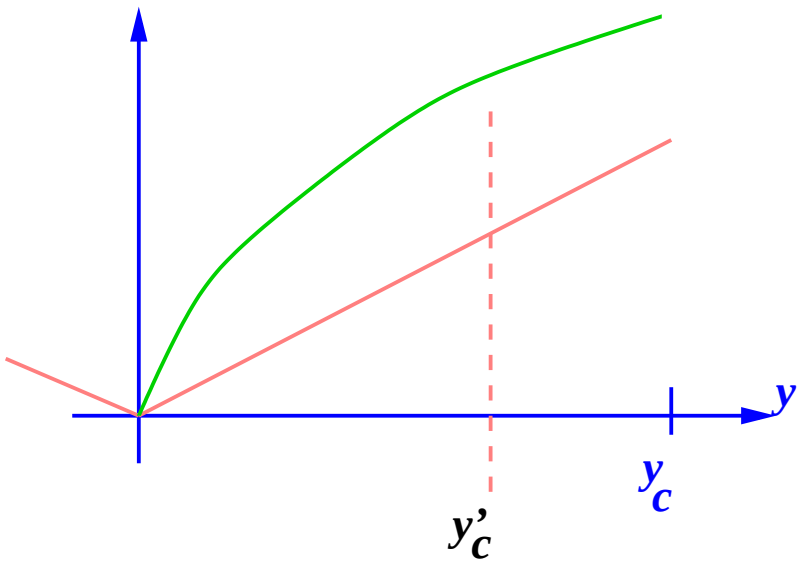
$$S = \int d^5x\,\sqrt{G}\,[-R-\Lambda] - \int_{y=0} d^4x\,\sqrt{-g}\,[V_0] - \int_{y=y_c} d^4x\,\sqrt{-g}\,[V_c]$$

and matter

$$S_B^A = \text{diag}\,(\rho, -p, -p, -p, 0)$$

- Solution

$$\begin{aligned} a^2 &= a_*^2 \cosh[2k(y-y_*)] \\ n^2 &= n_*^2 \sinh[2k(y-y_*)]\tanh[2k(y-y_*)] \\ b^2 &= 1 \end{aligned}$$

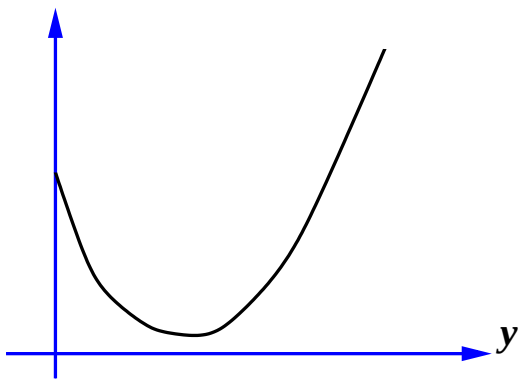


- Physics as before

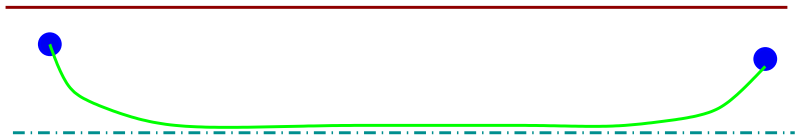
AdS_5 with AdS_4 sections

- Warp factor

$$A = \log\left(\cosh\frac{c-|y|}{L}\Big/\cosh\frac{c}{L}\right)$$



- Fixed-t geodesics at large Δx go close to $y = c$



- There is no brane located there, still distance scales for a bulk observer are as if measured along a $y = c$ brane
- The graviton is localized on the brane at $y = 0$
- The spectrum of massive excitations of the graviton is observed to be exponentially lighter by a brane observer than by a bulk observer, giving an amusing inverted hierarchy

Summary

- Effective action may be different for bulk/brane observers
- x for bulk observer determined by minimum distance
- Masses best determined by covariant approach