Local vs Global Duality in Hadronic Widths

Benjamín Grinstein, UCSD

Based on:

B. Grinstein, hep-ph/0106205	
B. Grinstein and R. Lebed	Phys.Rev.D57:1366(1998)
	Phys.Rev.D59:054022(1999)

Critique:

I. Bigi et al,	Phys.Rev.D59:054011(1999)
I.Bigi and N.Uraltsev	Phys.Lett.B457:163(1999)
	Phys.Rev.D60:114034(1999)

Other evidence:

G. Altarelli et al	Phys.Lett.B382:409(1996)
P. Colangelo et al	Phys.Lett.B409:417-424(1997)

Smearing, PQW (and the Theoretical Discovery of τ-leptons and b-quarks)

- Smear R:

$$\langle R\rangle(s,\Delta) = \frac{\Delta}{\pi}\int_0^\infty \frac{ds'\ R(s')}{(s-s')^2+\Delta^2}$$

$$R(s) \equiv \frac{\sigma_{e^+e^-\to hads}(s)}{\sigma_{e^+e^-\to\mu^+\mu^-}(s)} = \frac{12\pi s\sigma_{e^+e^-\to hads}(s)}{e^4}$$

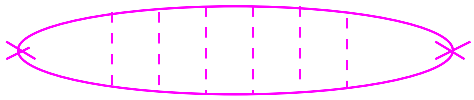
- Dispersion relation for vacuum polarization Π

$$\Pi(z) = \frac{1}{\pi}\int_0^\infty ds \frac{R(s)}{z-s}$$

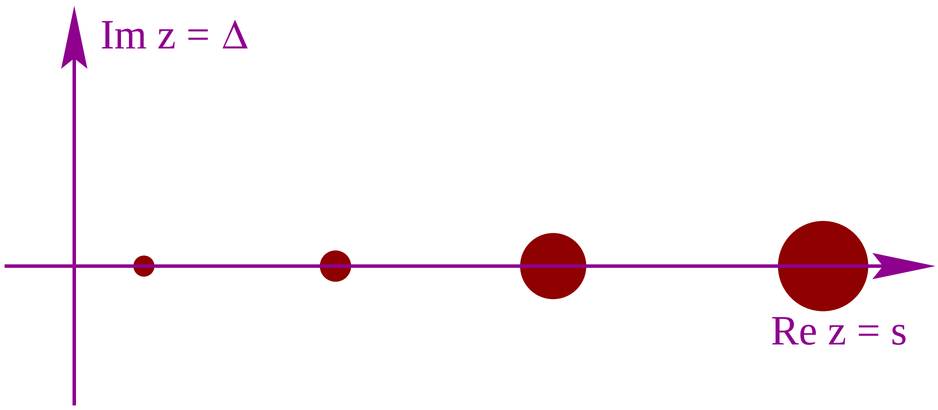
$\Rightarrow$

$$2i\langle R\rangle(s,\Delta) = \Pi(s+i\Delta) - \Pi(s-i\Delta)$$

- PQW reason:
  - IR singularities $\rightarrow$ perturbation theory fails

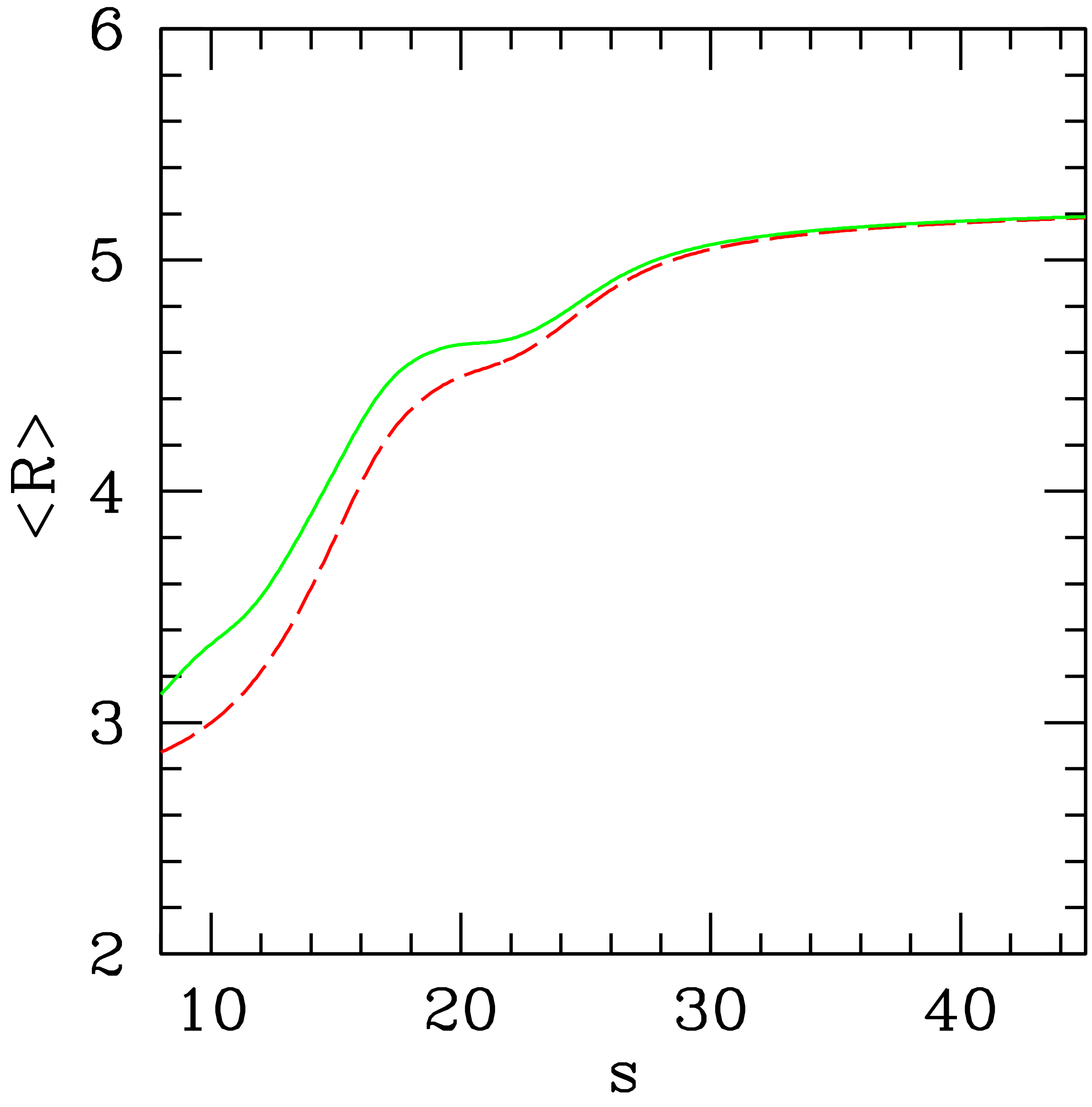


  - $\Delta \neq 0 \rightarrow$ prevents vanishing of propagator denominators
  - May break down at high orders of perturbation theory (very small fraction of Δ through each line)



  - Show this in QED, guess QCD too
  - Compare theory (parton) rates with experiment (after smearing)

Effect of including narrow resonances
$(J/\psi, \psi' \ldots \psi^{(v)})$ in smeared experimental curve



- - PQW

— PQW + narrow resonances

Duality in “B” Decays

In The t’Hooft Model

- t’Hooft Model: Large N QCD in $1+1$ dims
- Why?
  - Asymptotically Free
  - Confining
  - QCD-like spectrum
  - Soluble $\Rightarrow$ Lab for Theory
- Used previously for analogues of
  - $e^+e^- \to$ hadrons
  - DIS ($ep \to eX$)
  - Form factors
  - $+\cdots$

- Total (hadronic) width of heavy meson. Duality?

$$\Gamma \overset{?}{\sim} \mathrm{Im} \int d^4x \langle B|T(\mathcal{H}(x)\mathcal{H}(0))|B\rangle$$

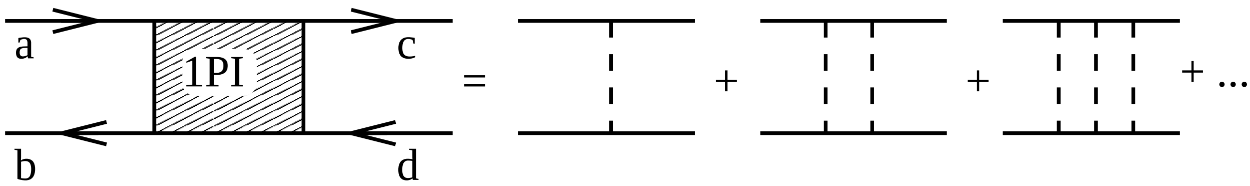
  - Large $m_b v$ flow $\Rightarrow$ HQET/OPE ?
  - If so: no $1/m_b$ corrections?
  - Cut over what variable? (eg, fake external momentum q with $q \to 0$?)
  - eg, Smear over m_b?

Elements of t'Hooft Model

- Lagrangian as in QCD

$$\mathcal{L} = -\frac{1}{4}\mathrm{Tr}\, F_{\mu\nu}F^{\mu\nu} + \sum_a \bar{\psi}_a \left(\gamma^\mu (i\partial_\mu - gA_\mu) - m_a\right)\psi_a$$

- Coupling g has mass dimension 1
  - Super-renormalizable $\Rightarrow$ asymptotic freedom
  - $g\sqrt{N}$ analogue of Λ_{QCD}
- Large N, $g^2 N$ fixed $\Rightarrow$ planar diagrams



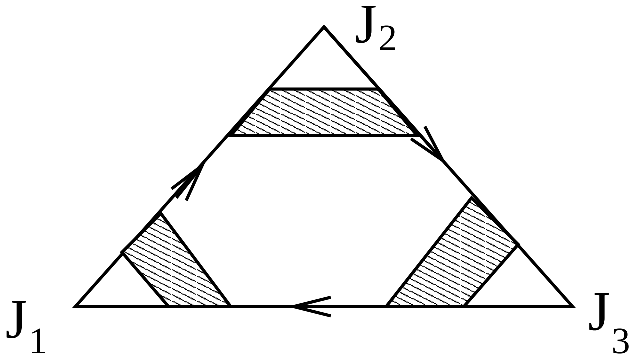
$$= \sum_n \frac{\phi_n(x)\phi_n(y)}{s - \mu_n^2},$$

$$\begin{aligned} s &= (p_a + p_b)^2 \\ x &= (p_a)_- / (p_a + p_b)_- \\ y &= (p_c)_- / (p_a + p_b)_- \end{aligned}$$

and ϕ_n from 'tHooft equation

$$\mu_n^2 \phi_n(x) = \left(\frac{m_1^2}{x} + \frac{m_2^2}{1-x}\right)\phi_n - \int_0^1 dy\, \phi_n(y)\ \mathrm{Pr}\,\frac{1}{(y-x)^2}$$

- All Green functions in terms of this, eg, $\langle T(J_1 J_2 J_3)\rangle =$



- No cuts, only poles $\Rightarrow \Gamma$ only from two body decays

$$\Gamma = \sum_{n,m} \Gamma(B \to \pi_n \pi_m)$$

B: lightest $\bar{Q}q$ state

π_n: tower of $\bar{q}q$ states

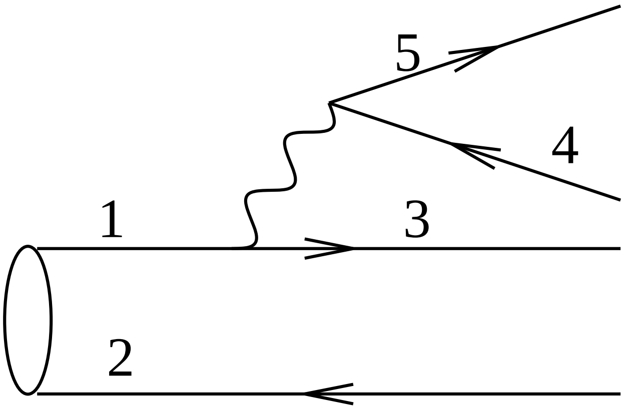
- In D dims

$$d\Gamma = \frac{|\mathbf{p}|^{D-3}}{(2\pi)^{D-2}\, 8M^2} |\mathcal{M}|^2 d\Omega$$

$\Rightarrow$ for $D = 2 = 1 + 1$ phase space diverges at threshold

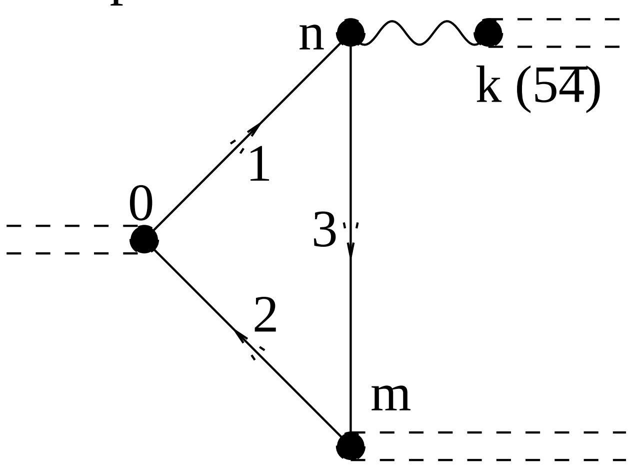
Computing Widths

- Parton model:



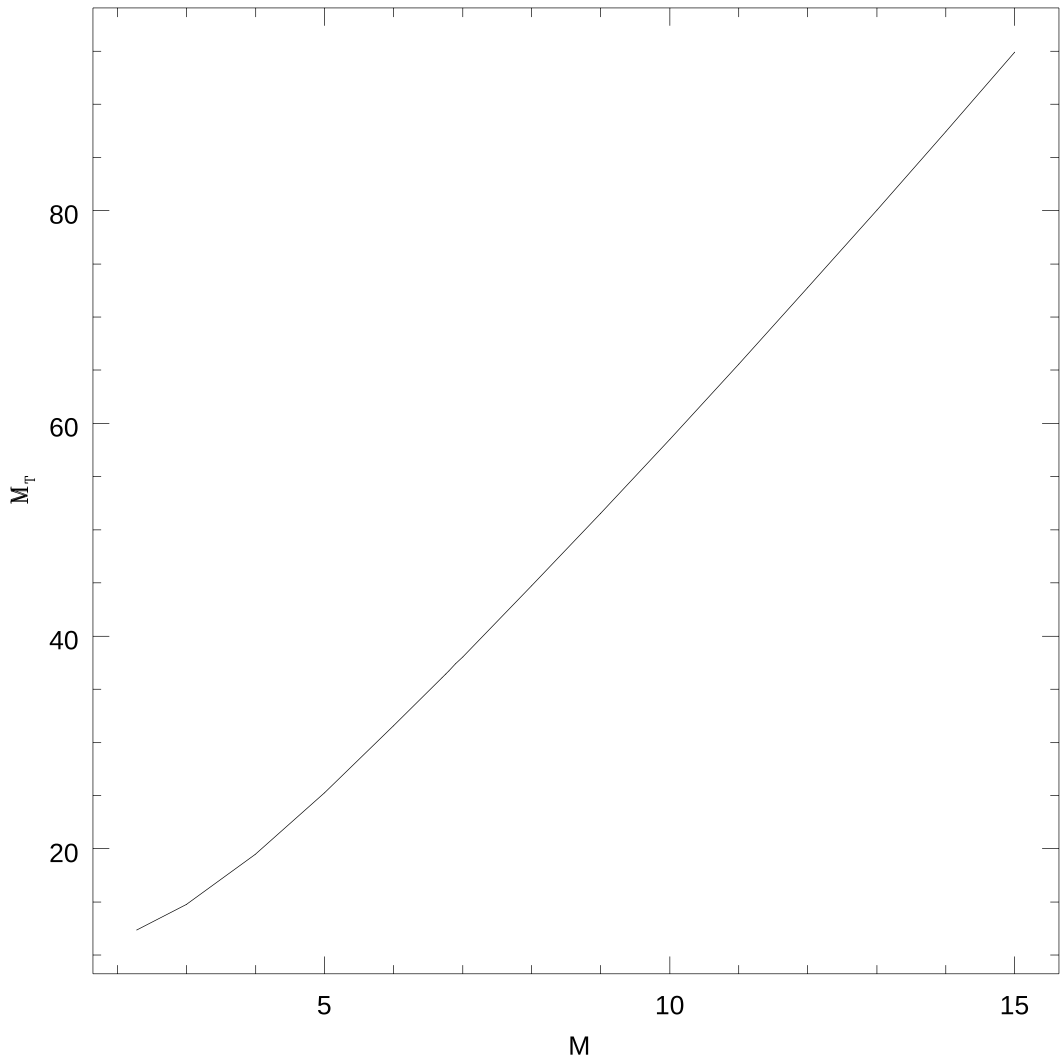
$$m_1 = m_Q = M$$
$$m_2 = m_3 = m_4 = m_5 = m_q$$
$$m_q \leq \Lambda_{\mathrm{QCD}} = g\sqrt{N} \ll m_Q$$

- Exact:
  - Compute heavy-light ϕ_0 (B) and mass M_B
  - ... and light-light ϕ_n's and masses μ_n
  - Given m_Q determine (n, m) so $\mu_n + \mu_m < M$
  - Compute

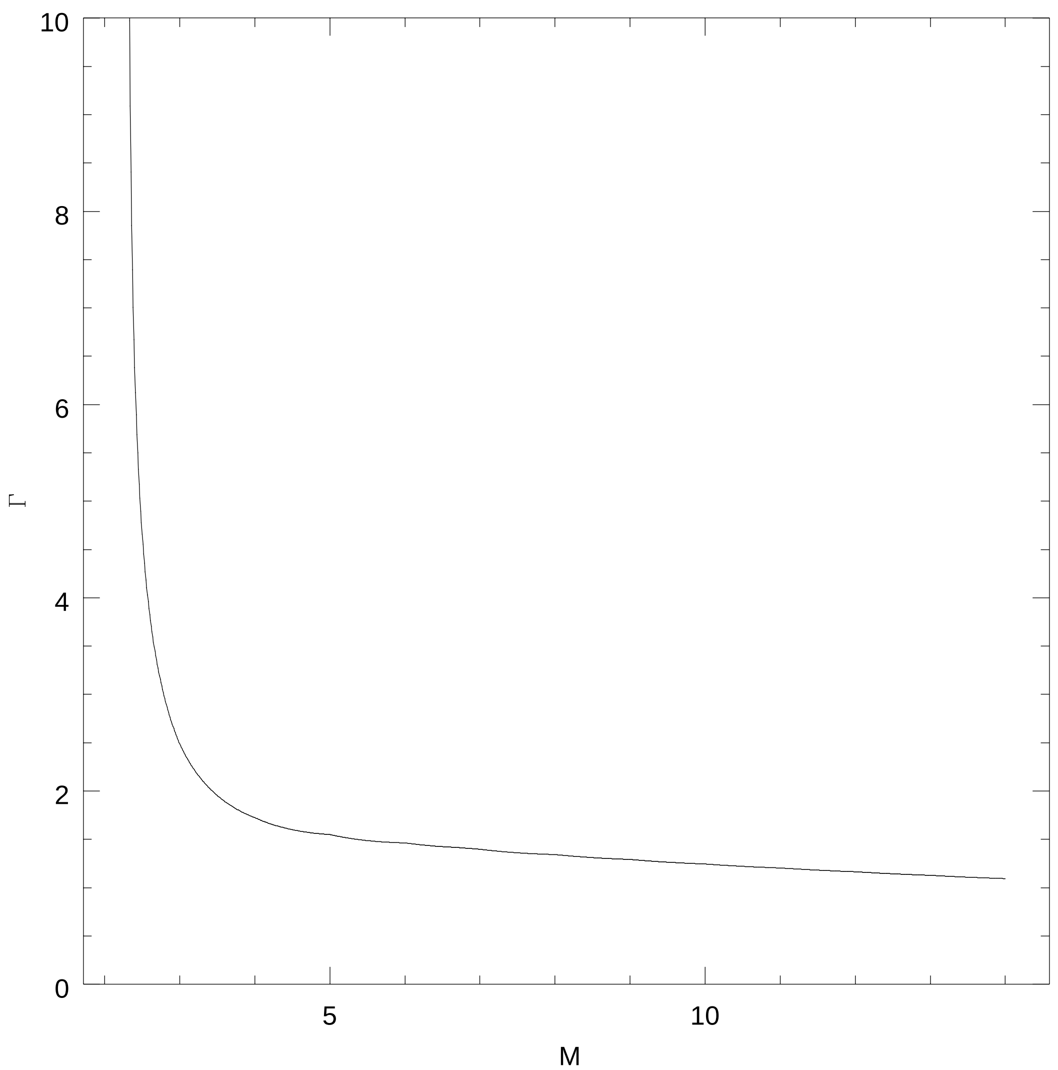


Results

- ϕ_n's and integrals computed numerically
- $g^2 N/\pi = 1$ sets units
- $m_q = 0.56$, $M = 2.28 \to 15.00$
- Decay amplitudes $\mathcal{M}$ smooth in M figure
- Each partial Γ diverges at threshold figure
- Sum over final states figures
- Compare with parton (dual rate) figure
- Conclude: Looks good!

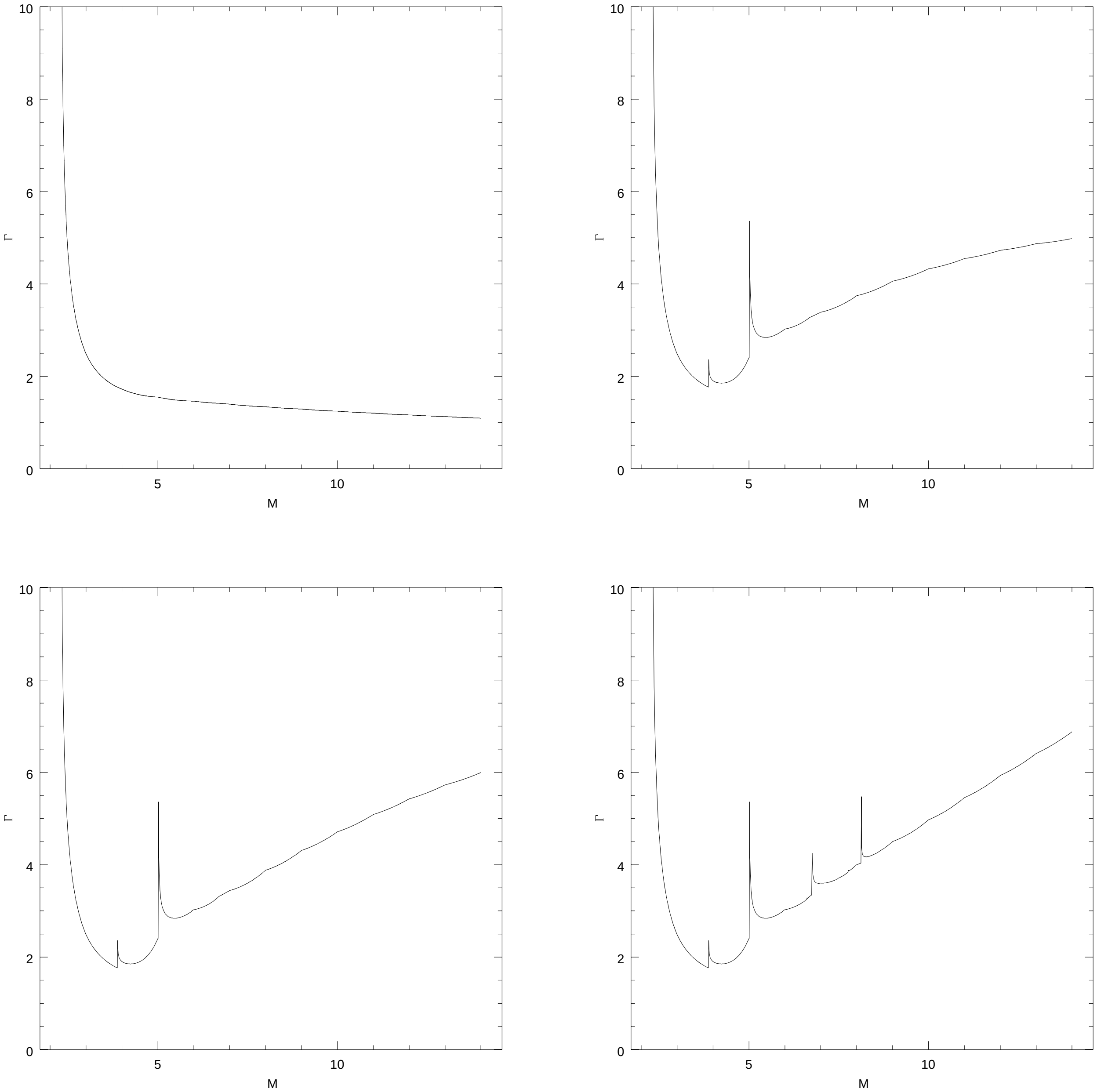


Weak decay amplitude $\mathcal{M}_T$ for the exclusive decay to the lowest mode $B \to \pi_0\pi_0$, as a function of heavy quark mass M, with light quark mass $m_q = 0.56$. The overall factor $2\sqrt{2/\pi}\, G_F V_{31} V_{45}^* (c_V^2 - c_A^2)$ in the amplitude is suppressed for convenience.

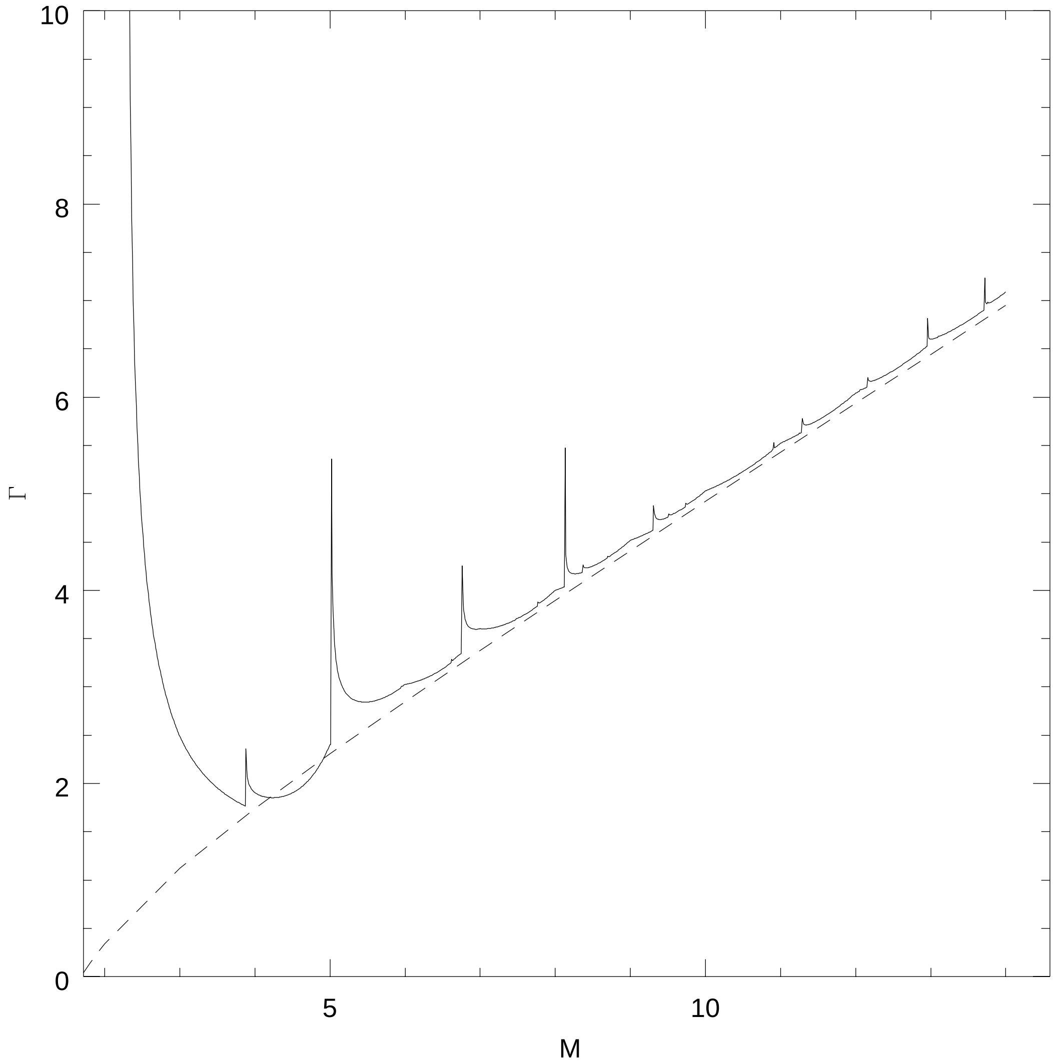


The full decay width as a function of heavy quark mass M, with light quark mass $m_q = 0.56$, including only the exclusive mode with the lowest threshold value ($\pi_0\pi_0$). The scale is the same as in the previous transparency

How resonances stack up

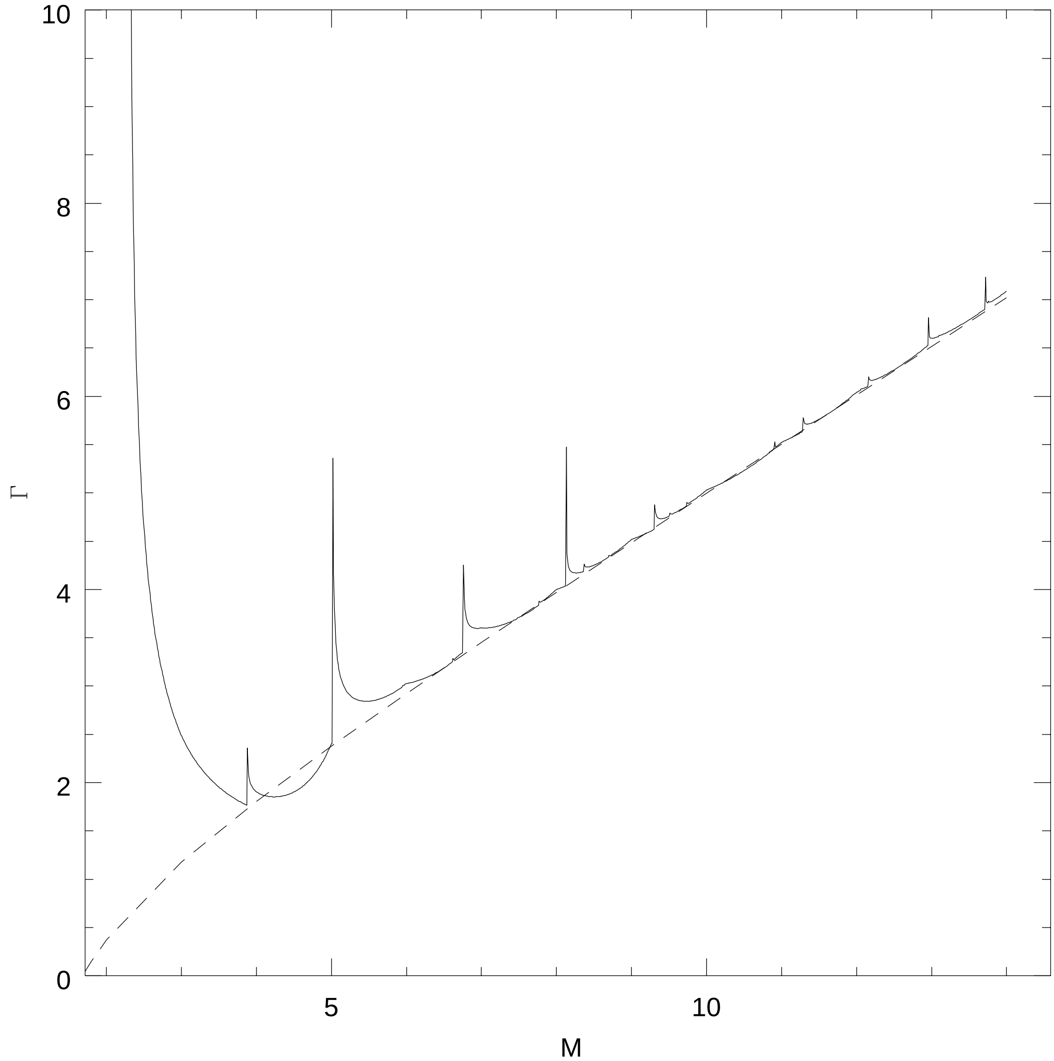


Contributions of first 1, 3, 5 and 11 channels to Γ



The full decay width for the sum of exclusive modes in the decay $B \to \pi_n \pi_m$ as a function of heavy quark mass M, with light quark mass $m_q = 0.56$. The dashed line is the tree-level parton result. The overall factor $8G_F^2|V_{31}V_{45}^*|^2(c_V^2 - c_A^2)^2/\pi$ in the width is suppressed for convenience.

- Replace $\Gamma_{\rm part}(M) \to \Gamma_{\rm part}(M) \cdot (1 + 0.15/M)$



- Looks BETTER!

Global vs Local Duality?

- "OPE" $\Rightarrow$ no $1/M$ corrections.
- Numerics or real?
- NEW RESULTS
  - Use same data
  - Smear over M (both Γ and Γ_{part}).
  - Wise: if larger weight under peaks at lower M $\Rightarrow$ turn $1/M$ into $1/M^2$

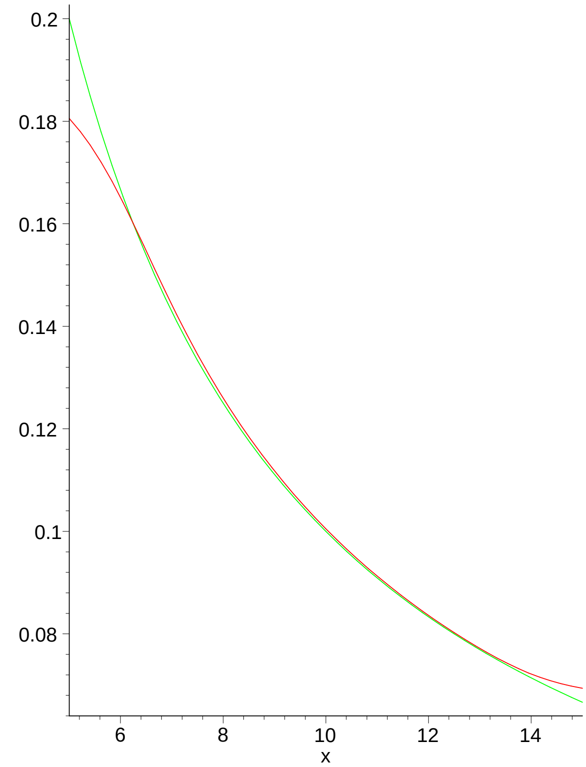
Define gaussian smearing

$$\langle f(M)\rangle = \frac{\int_{M_{\min}}^{M_{\max}} dx x^n e^{-(x-M)^2} f(x)}{\int_{M_{\min}}^{M_{\max}} dx x^n e^{-(x-M)^2}}$$

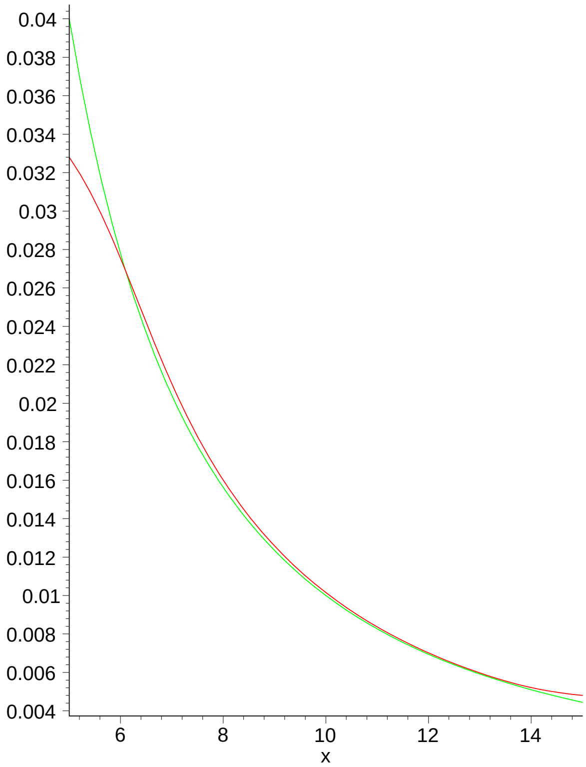
with $M_{\min} = 2.28$ and $M_{\max} = 15.00$

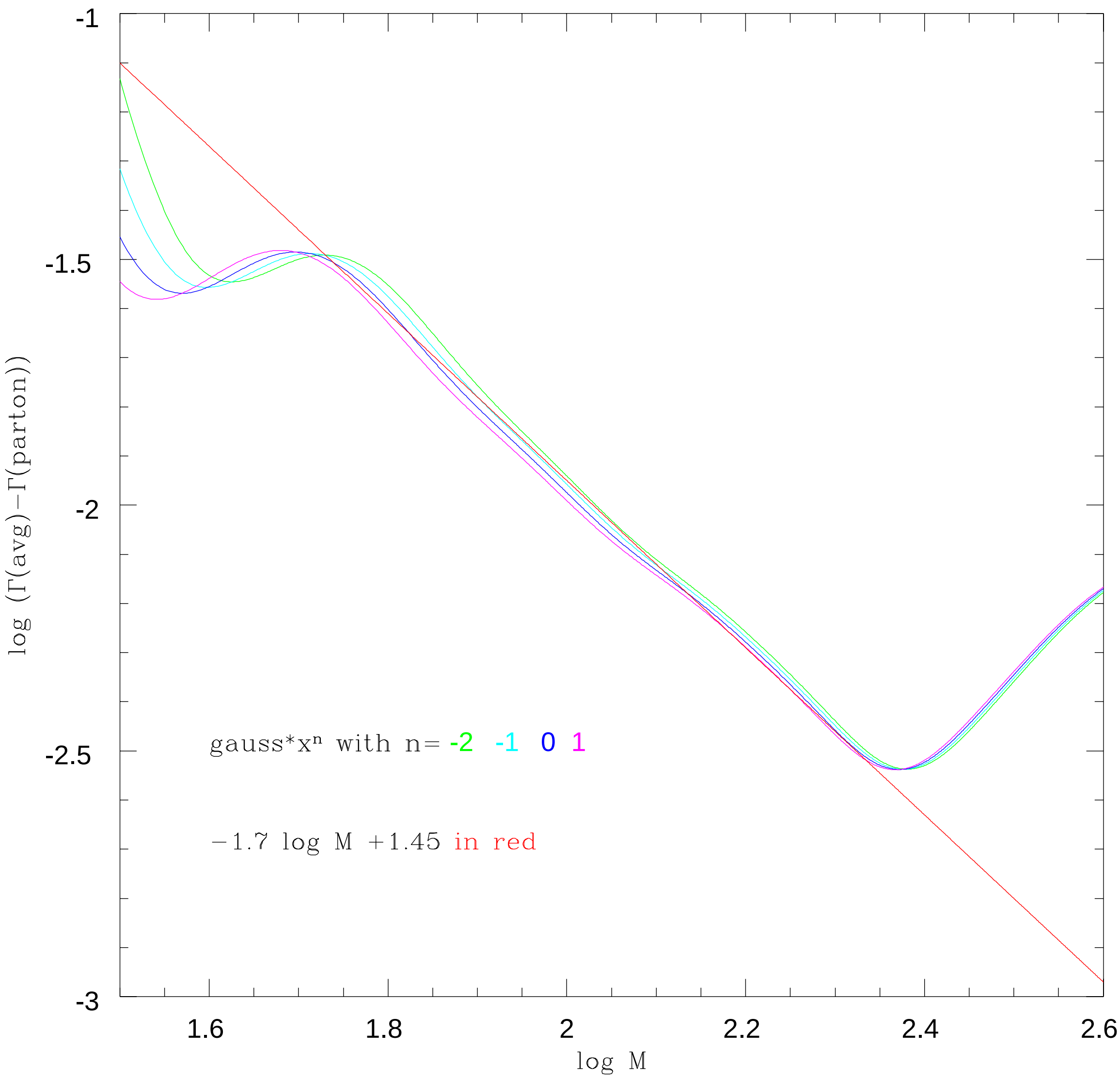
Smearing distorts endpoints

$1/x$
$\langle 1/x \rangle$



$1/x^2$
$\langle 1/x^2 \rangle$

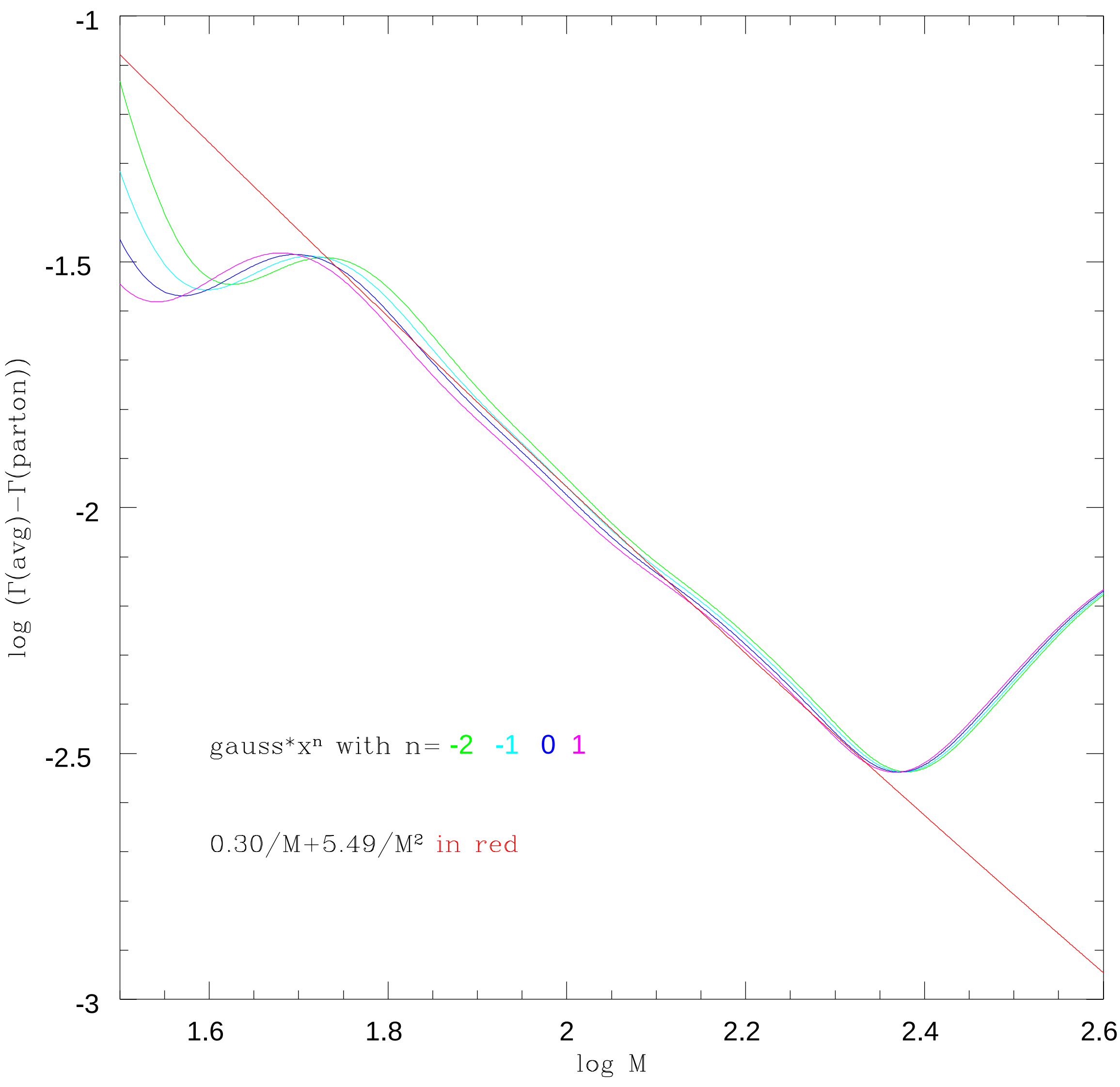




- Linear eye-fit $\ln(\Gamma - \Gamma_{\text{part}}) = -1.7 \ln M + 1.45$
- Fit $\exp(-1.7 \ln M + 1.45)$ in $\ln M \in [1.8, 2.3]$ to

$$M\left[\frac{a}{M} + \frac{b}{M^2} + \frac{c}{M^3}\right]$$

$\Rightarrow a \ll 1 \qquad b \approx 0.3 \qquad c \approx 5.5$



There is no $1/M$ correction!

Lessons

- $1/M$ in local data unlikely to be "numerical"
- There is a $1/M$ correction locally
- There is no $1/M$ correction after smearing $\Rightarrow$ "OPE" works for smeared quantities, but not locally
- We don't know how to justify this (continue into complex mass?)
- What about us? In 4D ...
  - Phase space at threshold vanishes
  - Many more resonances (and spin)
  - $\Rightarrow$ expect $\Gamma - \Gamma_{\rm part}$ has smooth oscillations of amplitude $\Gamma_{\rm part} \times 1/m$

Cannot exclude significant $1/m$ correction at actual value of B mass