

# Precision determination of $|V_{ub}|$

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## Difficulties with $V_{ub}$

Inclusive determination of  $V_{ub} \Leftrightarrow$  model independent

Stringent cuts required to suppress overwhelming background from  $b \rightarrow c$  decay

Possible cuts to reject background

- Cut on charged lepton energy  $E_\ell > \frac{m_B^2 - m_D^2}{2m_B}$   
Argus (1991)  
CLEO (1993)
- Cut on hadronic invariant mass  $m_X < m_D$   
Falk, Ligeti, Wise (1997)  
Bigi, Dikeman, Uraltsev (1998)  
ALEPH, L3 (1998)
- Cut on leptonic invariant mass  $q^2 > (m_B - m_D)^2$   
CB, Luke, Ligeti (2000)

Restricted regions of phase space  $\Leftrightarrow$  OPE convergence problems

**Find optimal cut (combinations of cuts)**

# Kinematics

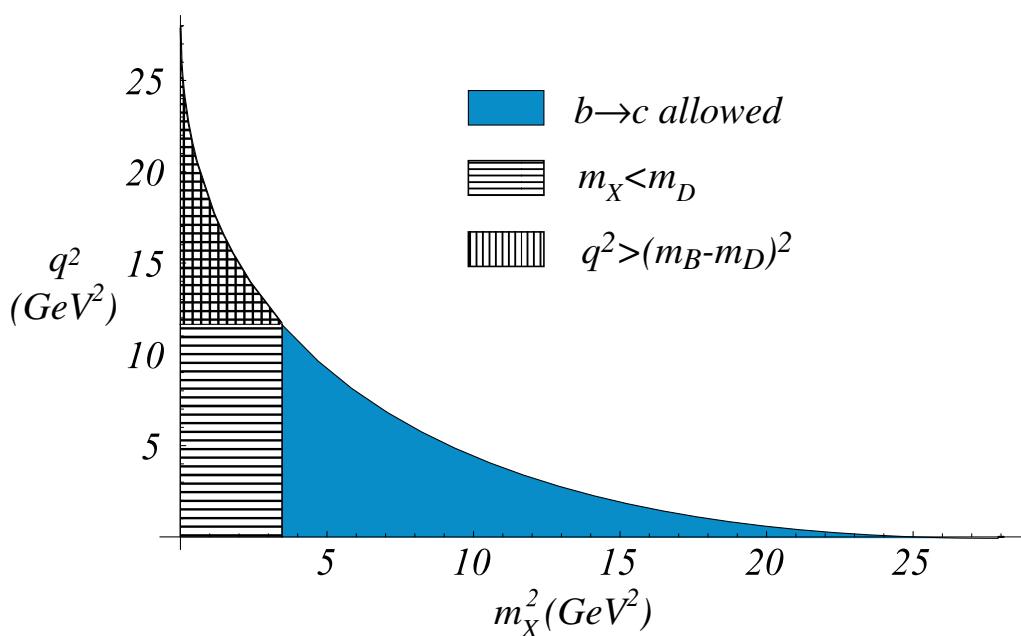
OPE converges only for sufficiently inclusive observables

$\Rightarrow$  need  $m_X \gg \Lambda_{\text{QCD}}$

Expansion contains twist terms  $\sim \frac{\Lambda_{\text{QCD}} E_X}{m_X^2}$

$\frac{m_X^2}{E_X} \gg \Lambda_{\text{QCD}} \Rightarrow$  standard OPE converges

$\frac{m_X^2}{E_X} \sim \Lambda_{\text{QCD}} \Rightarrow$  twist expansion required



## Combination of cuts

- The  $q^2$  cut includes subset of events of the  $m_X$  cut
- Advantages and disadvantages in  $q^2$  and  $m_X$  spectrum are reversed

$q^2$ cut	$m_X$ cut
small # of events	large # of events
large $1/m^3$ effect	small $1/m^3$ effects
no model dep.	large model dep.

- Interpolating between both cuts gives  $V_{ub}$  with
  - large # of events
  - small  $1/m^3$
  - small model dependence

## Precision determination of $V_{ub}$

## The calculation

Relating  $|V_{ub}|$  to the partially integrated rate

$$\int \frac{d\Gamma}{d\hat{q}^2 d\hat{s}} d\hat{q}^2 d\hat{s} \equiv \frac{G_F^2 |V_{ub}|^2 (4.7 \text{ GeV})^5}{192\pi^3} \underbrace{G(q_{\text{cut}}^2, m_{\text{cut}})}_{1.21 \times (\text{fraction of events})}$$

Contributions to the differential rate

$$\frac{d\Gamma}{d\hat{q}^2 d\hat{s}} = \delta(\hat{s}) \left[ \frac{d\Gamma_0}{d\hat{q}^2} + \frac{1}{m_b^2} \frac{d\Gamma_2}{d\hat{q}^2} \right] + \frac{\alpha_s}{\pi} X(\hat{q}^2, \hat{s}) + \frac{\alpha_s^2 \beta_0}{\pi^2} Y(\hat{q}^2, \hat{s})$$

To estimate effect of structure function

$$\delta(\hat{s}) \rightarrow \begin{cases} \int dk_+ f(k_+) \delta(\hat{s} + (1 - \hat{q}^2) \hat{k}_+), & \hat{q}^2 < (1 - \hat{m}_{\text{cut}})^2 \\ \delta(\hat{s}), & \text{otherwise.} \end{cases}$$

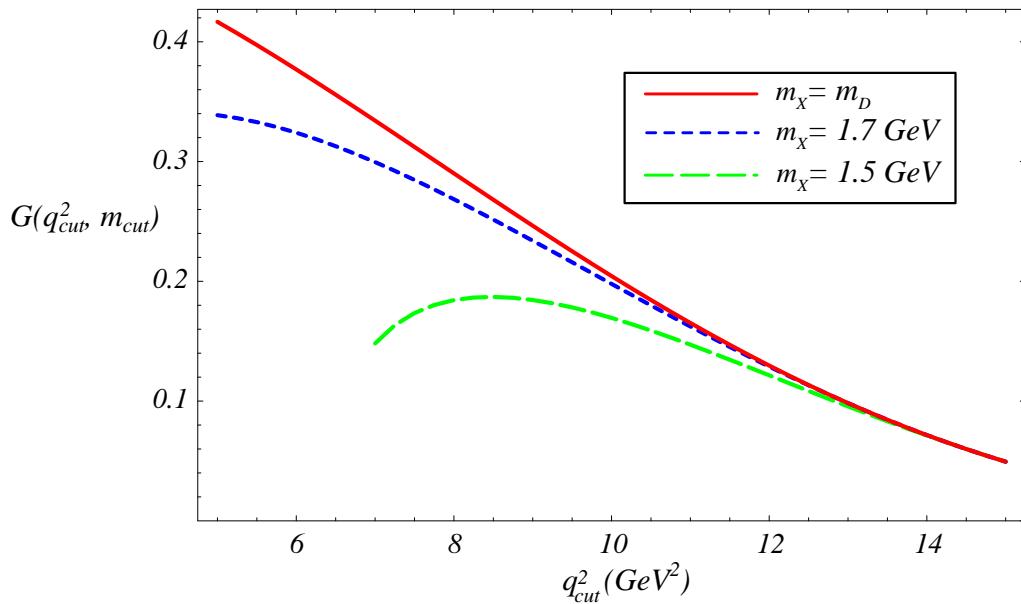
Determine structure function from  $B \rightarrow X_s \gamma$

Neubert (1995)

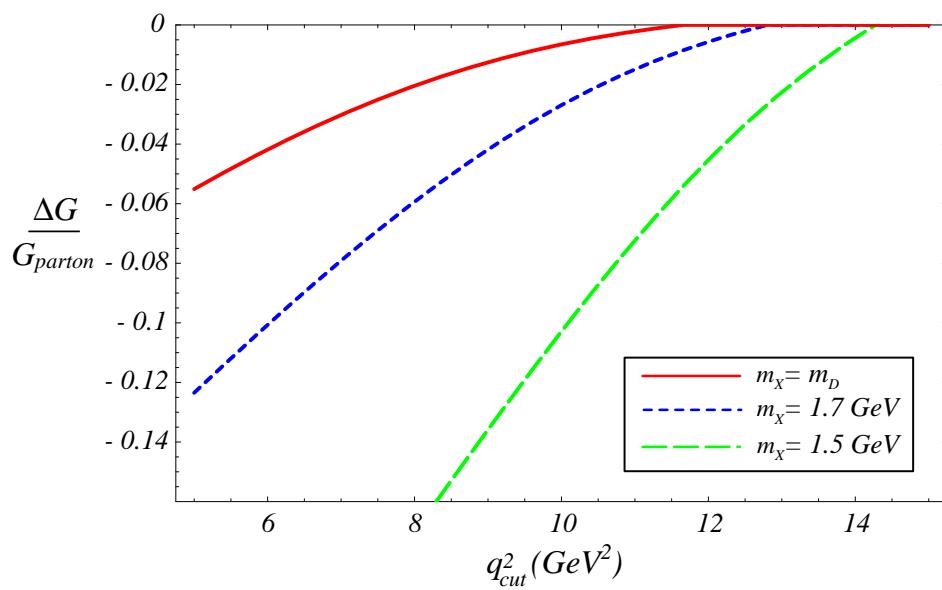
$$f(k_+) = \frac{1}{2K\Gamma^\gamma} \frac{d\Gamma^\gamma}{dE_\gamma} \Big|_{E_\gamma = \frac{m_b + k_+}{2}} + \mathcal{O}\left(\frac{1}{m_b}\right)$$

# Results

Three different values of  $m_{cut}$

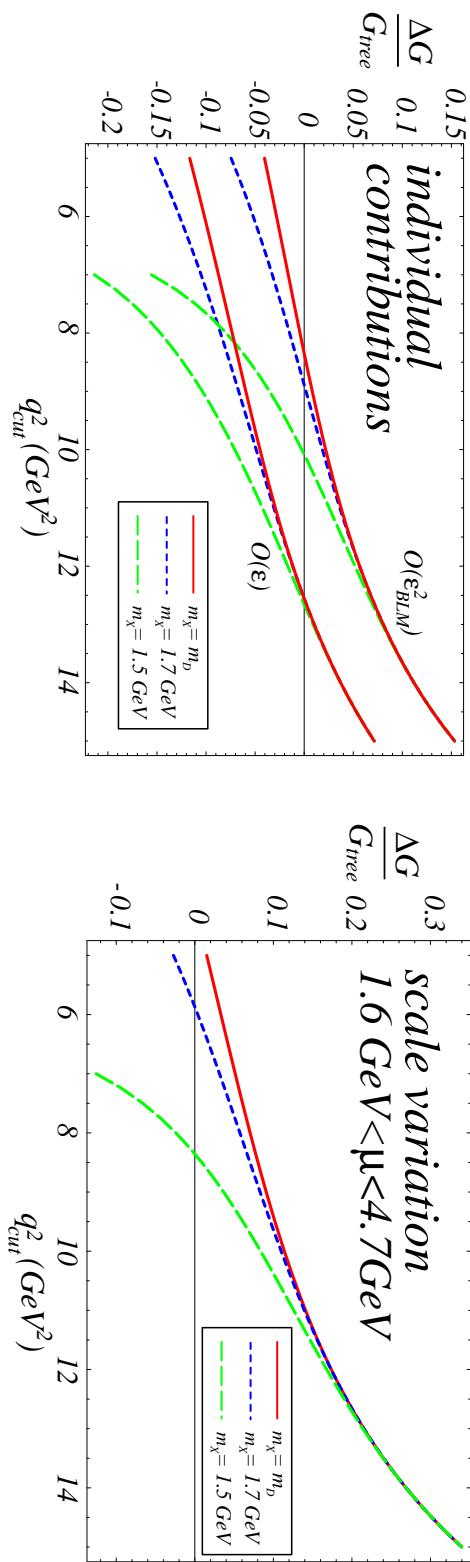


Effect of the structure function

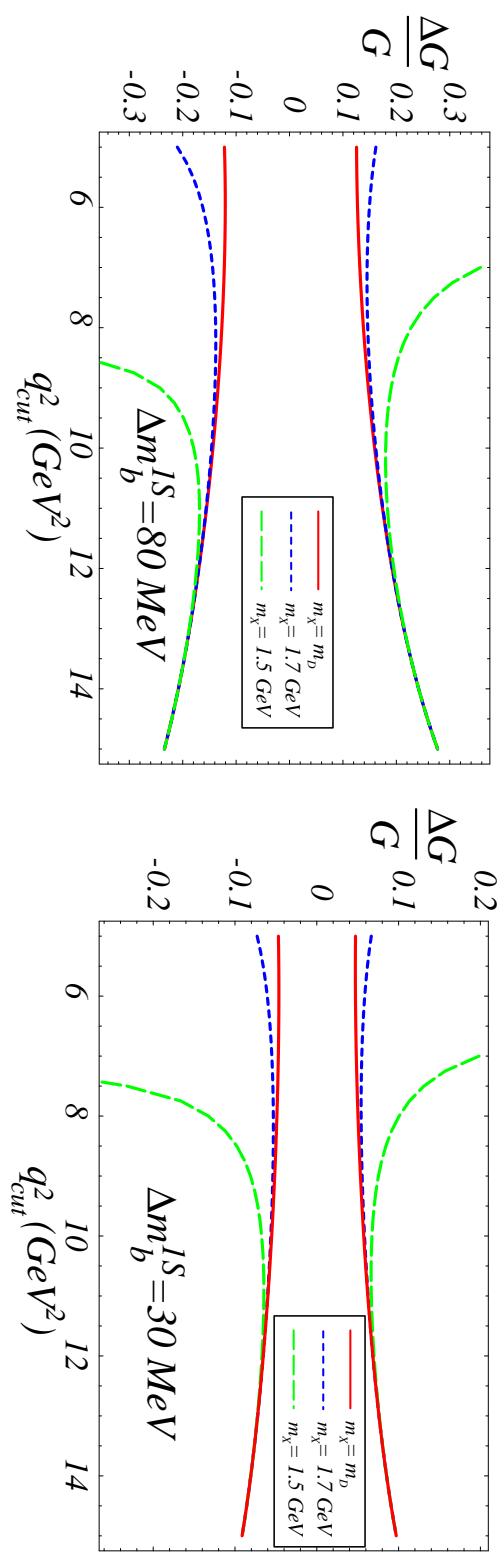


## Error Estimates

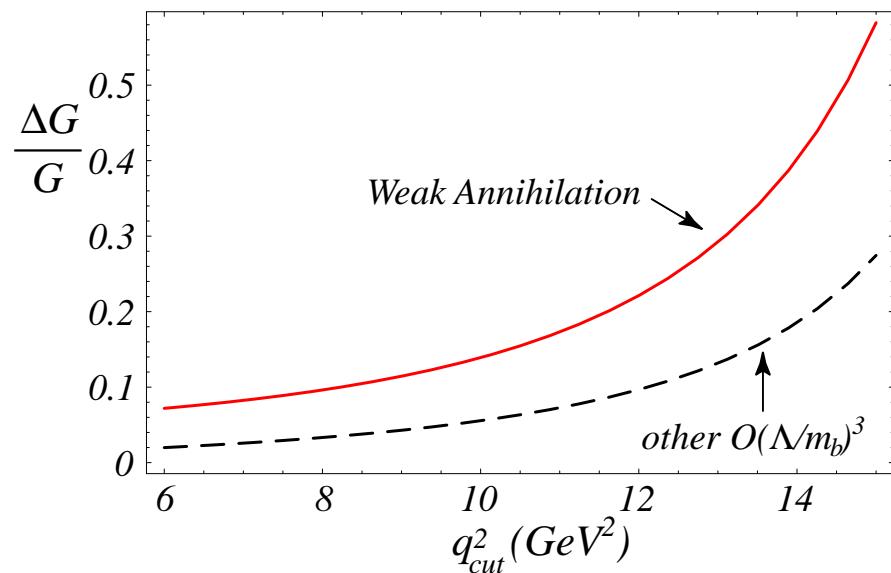
### Perturbative Uncertainties



## Uncertainties due to $b$ quark mass



Nonperturbative  $\mathcal{O}\left(\frac{1}{m_b^3}\right)$  uncertainties



## Final Results

$$(\Delta|V_{ub}| = \frac{\Delta G}{2})$$

Cuts on $(q^2, m_X^2)$	$G$	$\Delta_f G$	$\Delta_{\alpha_s} G$	$\Delta_{m_b} G$ $\pm 80/30 \text{ MeV}$	$\Delta_{1/m^3} G$	$\Delta G$
Combined cuts						
6 $\text{GeV}^2, 1.86 \text{ GeV}$	<b>0.38</b>	-4%	4%	13%/5%	6%	<b>15%/9%</b>
8 $\text{GeV}^2, 1.7 \text{ GeV}$	0.27	-6%	6%	15%/6%	8%	18%/12%
11 $\text{GeV}^2, 1.5 \text{ GeV}$	0.15	-7%	13%	18%/7%	16%	27%/22%
Pure $q^2$ cuts						
$(m_B - m_D)^2, m_D$	<b>0.14</b>	--	15%	19%/7%	18%	<b>30%/24%</b>
$(m_B - m_{D^*})^2, m_{D^*}$	0.17	--	13%	17%/7%	14%	26%/20%

- combined cuts sensitive up to 45% events
- uncertainties less than half of pure  $q^2$  cut

Uncertainties on  $|V_{ub}|$  5-10%

## Conclusions

General strategy to determine  $|V_{ub}|$ :

- $m_{\text{cut}}$  as large as possible, keeping background from  $b \rightarrow c$  under control
- for given  $m_{\text{cut}}$ , make  $q_{\text{cut}}$  as low as possible, keeping contribution from  $f(k_+)$  and perturbative uncertainties under control

Optimal value of cuts determined by experimental analysis

**Uncertainties on  $|V_{ub}|$  5-10%**