Overview of deeply virtual Compton scattering

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- Kinematics of DVCS
- DVCS and generalized parton distributions
- Azimuthal angular dependence
- Measurements of Compton form factors
- Beyond leading order predictions
- Summary and conclusions

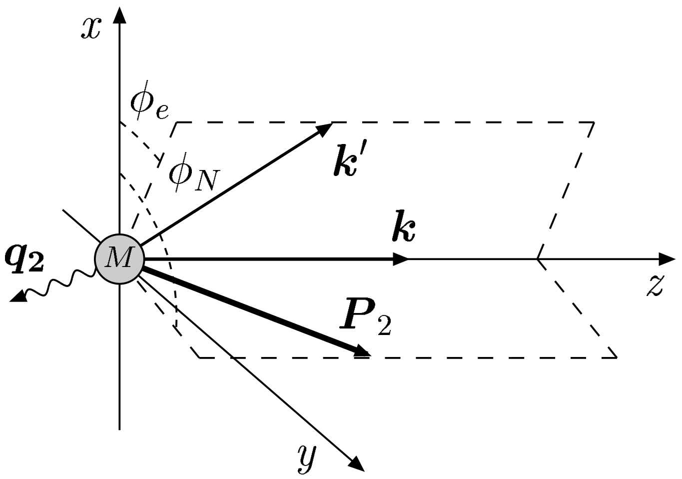
In collaboration with A.V. Belitsky, A. Kirchner, L. Niedermeier, A. Schäfer

Kinematics of DVCS

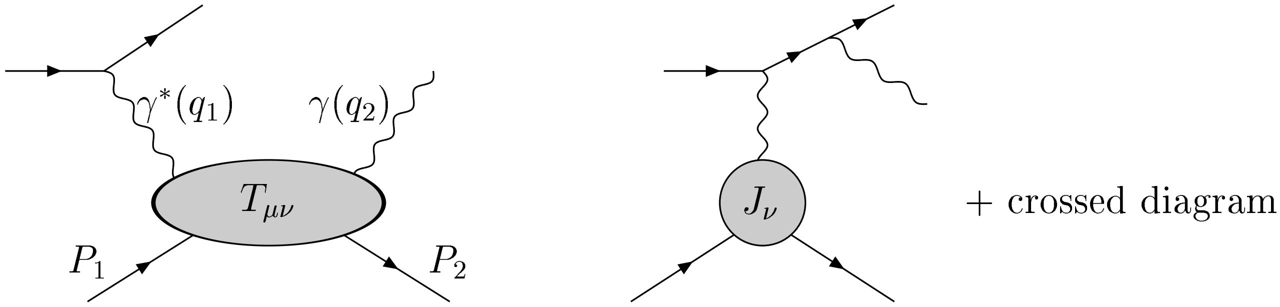
Cross section for $e(k, \lambda) N(P_1, S_1) \to e(k', \lambda') N(P_2, S_2) \gamma(q_2, \Lambda)$:

$$\frac{d\sigma}{dx_{\mathrm{B}} dy d|\Delta^2| d\phi_e d\phi_N} = \frac{\alpha^3 x_{\mathrm{B}} y}{16\,\pi^2\,\mathcal{Q}^2} \left(1 + \frac{4M^2 x_{\mathrm{B}}^2}{\mathcal{Q}^2}\right)^{-1/2} \left|\frac{\mathcal{T}}{e^3}\right|^2,$$

where $x_{\mathrm{B}} = \frac{\mathcal{Q}^2}{2P_1 \cdot q_1} \approx \frac{2\xi}{1+\xi}$, $\mathcal{Q}^2 = -q_1^2$, $y = \frac{P_1 \cdot q_1}{P_1 \cdot k}$, $\Delta^2 = t$, and:



γ-production contains both DVCS and Bethe-Heitler process



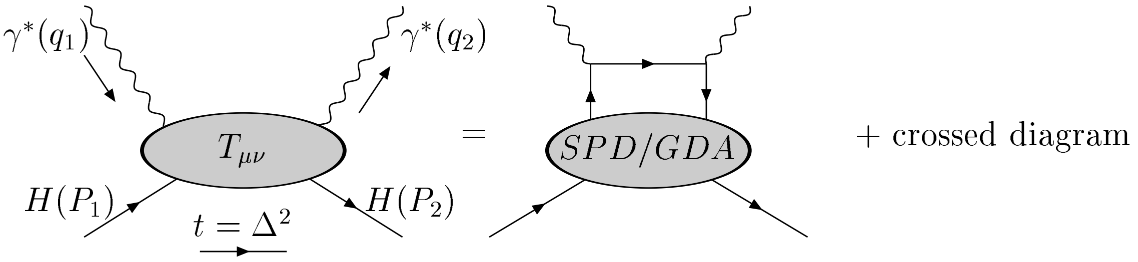
The squared amplitude is decomposed in three parts

$$|\mathcal{T}|^2 = \sum_{\lambda', S_2, \Lambda} \left\{ |\mathcal{T}_{VCS}|^2 + |\mathcal{T}_{BH}|^2 \pm \mathcal{I} \right\} \quad \begin{cases} + \ e^-\text{-beam} \\ - \ e^+\text{-beam} \end{cases}$$

with the interference term $\mathcal{I} = \mathcal{T}_{VCS} \mathcal{T}_{BH}^* + \mathcal{T}_{VCS}^* \mathcal{T}_{BH}$.

Calculation of DVCS amplitude

Generalized parton distributions (GPDs) and generalized distribution amplitudes (GDAs) appear in virtual two-photon processes:



Generalized Bjorken limit (light-cone-dominated region):

$$\text{large scale:} \quad Q^2 = -q^2 = -\frac{(q_1+q_2)^2}{4} \to \infty,$$

$$\text{fixed variables:} \quad t \equiv \Delta^2 = (P_2 - P_1)^2, \; P^2 = (P_1 + P_2)^2,$$

$$\text{scaling variables:} \quad \xi = \frac{Q^2}{q \cdot P}, \quad \eta = \frac{q \cdot \Delta}{q \cdot P}.$$

Amplitudes in the generalized Bjorken region

Employing the operator product expansion for the hadronic tensor

$$T_{\mu\nu}(q, P_1, P_2) = i \int d^4x e^{ix\cdot q} \langle P_2, S_2 | T j_\mu\Big(\frac{x}{2}\Big)\, j_\nu\Big(-\frac{x}{2}\Big)\, | P_1, S_1 \rangle,$$

provides the amplitudes for quite different two-photon processes [D. M, D. Robaschik, B. Geyer, F.M.-Dittes, J. Hořejši, Fortschr. Phys. 42 (1994) 101]:

DIS	$\Delta = 0$	$\eta = 0$
DVCS	$q_2^2 = 0$	$\eta \approx -\xi \approx -x_{\rm B}/(2 - x_{\rm B})$
$\gamma^{(*)} N \to N l^+ l^-$	$q_2^2 > 0$	$\eta \approx \cos\theta_{\rm Br} = \mathbf{P}_1\mathbf{Q}/\lvert\mathbf{P}_1\rvert\lvert\mathbf{Q}\rvert$, Breit–frame
$\gamma^*\gamma^{(*)} \to HH$		$\eta \approx \cos\theta_{\rm cm} = \mathbf{P}_1\mathbf{Q}/\lvert\mathbf{P}_1\rvert\lvert\mathbf{Q}\rvert$, cm–frame
$\gamma^*\gamma^{(*)} \to M$	$P_1 = 0$	$\eta = 1$

Leading order result: [D. M. et al.; X. Ji; A.V. Radyushkin.]

$$\mathcal{A}^{\rm LO} = \int_{-1}^{1} dx \left(\frac{1}{\xi - x - i0} \pm \frac{1}{\xi + x - i0} \right) q(x, \eta, \Delta^2, \mu^2; S_1, S_2),$$

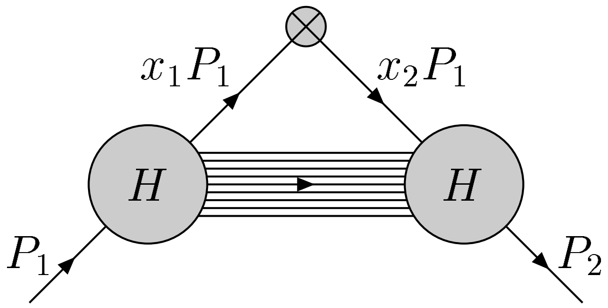
$$q(x, \eta, \Delta^2, \mu^2; S_1, S_2) = \int \frac{d\kappa}{2\pi} e^{i\kappa x P_+} \langle P_2 S_2 | \phi(-\kappa n) \phi(\kappa n)_{|\mu^2} | P_1 S_1 \rangle.$$

Parton interpretation

momentum fractions of partons:

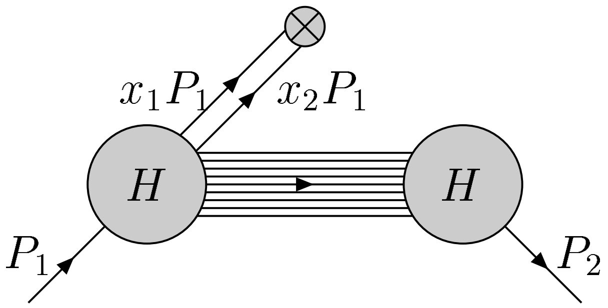
$x_1 = \dfrac{x + \eta}{1 + \eta}$ (incoming parton), $\qquad x_2 = \dfrac{x - \eta}{1 + \eta}$ (outgoing parton),

‘inclusive’ region



$\eta < x$, i.e. $x_1, x_2 > 0$,
$-\eta > x$, i.e. $x_1, x_2 < 0$.

‘exclusive’ region



$-\eta < x < \eta$,

The spin dependence can be parametrized by form factors and read for a spin-1/2 target [X. Ji Phys. Rev. D55 (1997) 7114]

$$\begin{aligned}
{}^{i}q^{V} &= \bar{U}(P_2, S_2)\gamma_+ U(P_1, S_1) H_i(x, \eta, \Delta^2, Q^2) \\
&+ \bar{U}(P_2, S_2) \frac{i\sigma_{+\nu}\Delta^\nu}{2M} U(P_1, S_1) E_i(x, \eta, \Delta^2, Q^2), \\
{}^{i}q^{A} &= \bar{U}(P_2, S_2)\gamma_+\gamma_5 U(P_1, S_1) \widetilde{H}_i(x, \eta, \Delta^2, Q^2) \\
&+ \bar{U}(P_2, S_2) \frac{\gamma_5}{2M} U(P_1, S_1) \widetilde{E}_i(x, \eta, \Delta^2, Q^2).
\end{aligned}$$

Azimuthal angular dependence

Reference frame

The choise $q_1 = (q_1^0, 0, -|q_1^3|)$ ensures a clear separation of twist-two and -three observables.

Azimuthal angular dependence of photon electroproduction:

The $\phi = \phi_N - \phi_e$ dependence arise from the contraction of leptonic and hadronic tensors, e.g.

$$k \cdot \Delta = \frac{-\mathcal{Q}^2}{2y}\Big\{1 + 2K\cos\phi + \mathrm{O}\left(\Delta^2/\mathcal{Q}^2\right)(\phi - \text{independent terms})\Big\},$$

where $K \propto \sqrt{\frac{-\Delta^2}{\mathcal{Q}^2}}\sqrt{1 - \frac{\Delta^2_{\min}}{\Delta^2}}$ and appears also in BH-propagators:

$$\mathcal{Q}^2\mathcal{P}_1 \equiv (k - q_2)^2 = \mathcal{Q}^2 + 2k \cdot \Delta \approx -\frac{\mathcal{Q}^2}{y}\left[1 - y - K\cos\phi + \cdots\right],$$

$$\mathcal{Q}^2\mathcal{P}_2 \equiv (k - \Delta)^2 = -2k \cdot \Delta + \Delta^2 \approx \frac{\mathcal{Q}^2}{y}\left[1 + 2K\cos\phi + \cdots\right].$$

The dependence on the azimuthal angle reads:

$$|\mathcal{T}^{\mathrm{BH}}|^2 = \frac{-1}{x_{\mathrm{B}}^2 y^2 \mathcal{P}_1(\phi)\mathcal{P}_2(\phi)\Delta^2}\sum_{n=0}^{2} c_n^{\mathrm{BH}}\cos(n\phi),$$

$$\mathcal{I} = \frac{\pm 1}{x_{\mathrm{B}} y^3 \mathcal{P}_1(\phi)\mathcal{P}_2(\phi)\Delta^2}\left\{c_0^{\mathcal{I}} + \sum_{n=1}^{3}\left[c_n^{\mathcal{I}}\cos(n\phi) + s_n^{\mathcal{I}}\sin(n\phi)\right]\right\},$$

$$|\mathcal{T}^{\mathrm{DVCS}}|^2 = \frac{1}{y^2\mathcal{Q}^2}\left\{c_0^{\mathrm{CS}} + \sum_{n=1}^{2}\left[c_n^{\mathrm{CS}}\cos(n\phi) + s_n^{\mathrm{CS}}\sin(n\phi)\right]\right\}.$$

Scaling with Q is generally predicted by the OPE [T. Gousset, M. Diehl, B. Pire, J.P. Ralston; A.V. Belitsky, D. M., A. Kirchner, A. Schäfer]:

$$c_0^{\mathcal{I}} \propto \frac{\Delta^2}{Q^2}\text{tw-2}, \begin{Bmatrix} c_1 \\ s_1 \end{Bmatrix}^{\mathcal{I}} \propto \frac{\Delta}{Q}\text{tw-2}, \begin{Bmatrix} c_2 \\ s_2 \end{Bmatrix}^{\mathcal{I}} \propto \frac{\Delta^2}{Q^2}\text{tw-3}, \begin{Bmatrix} c_3 \\ s_3 \end{Bmatrix}^{\mathcal{I}} \propto \frac{\Delta \alpha_s}{Q}(\text{tw-2})^{\text{GT}};$$

$$c_0^{\text{CS}} \propto (\text{tw-2})^2, \begin{Bmatrix} c_1 \\ s_1 \end{Bmatrix}^{\text{CS}} \propto \frac{\Delta}{Q}(\text{tw-2})(\text{tw-3}), \begin{Bmatrix} c_2 \\ s_2 \end{Bmatrix}^{\text{CS}} \propto \alpha_s(\text{tw-2})(\text{tw-2})^{\text{GT}}.$$

Twist-two predictions in terms of Compton form factors

The complete twist-two result is expressed in terms of four CFFs $\mathcal{F} = \left\{\mathcal{H}, \mathcal{E}, \widetilde{\mathcal{H}}, \widetilde{\mathcal{E}}\right\}$ [A.V. Belitsky, D. M., L. Niedermeier, A. Schäfer]

$$\mathcal{F}(\xi, \Delta^2|\mathcal{Q}^2) = \sum_{i=u,d,\dots} \int_{-1}^{1} dx\; C_i^{(\mp)}(\xi, x|\mathcal{Q}^2, \mu^2) F_i(x, \xi, \Delta^2|\mu^2),$$

where $F_i = \left\{H_i, E_i, \widetilde{H}_i, \widetilde{E}_i\right\}$ and $C_i^{(\mp)}$ are the coefficient functions.

Results are given in terms of 'universal' $\mathcal{C}$-functions, e.g. the interference term for unpolarized target reads

$$\begin{Bmatrix} c_1^{\mathcal{I}} \\ s_1^{\mathcal{I}} \end{Bmatrix}_{\text{unp}} = 8K \begin{Bmatrix} \text{Re} \\ \text{Im} \end{Bmatrix} \begin{Bmatrix} -(2-2y+y^2) \\ \lambda y(2-y) \end{Bmatrix} \mathcal{C}^{\mathcal{I}}_{\text{unp}}(\mathcal{F}),$$

$$\mathcal{C}^{\mathcal{I}}_{\text{unp}}(\mathcal{F}) = F_1\mathcal{H} + \frac{x_{\text{B}}}{2-x_{\text{B}}}(F_1+F_2)\widetilde{\mathcal{H}} - \frac{\Delta^2}{4M^2}F_2\mathcal{E},$$

For polarized target $[S = (0, \cos\Phi\sin\theta, \sin\Phi\sin\theta, \cos\theta)]$ one has six more observables ($\varphi = \phi_N - \Phi$):

$$\begin{Bmatrix} c_1^{\mathcal{I}} \\ s_1^{\mathcal{I}} \end{Bmatrix}_{\text{pol}} = \left[\begin{Bmatrix} c_1^{\mathcal{I}} \\ s_1^{\mathcal{I}} \end{Bmatrix}_{\text{TP}+} \cos\varphi + \begin{Bmatrix} c_1^{\mathcal{I}} \\ s_1^{\mathcal{I}} \end{Bmatrix}_{\text{TP}-} \sin\varphi\right] \sin\theta + \begin{Bmatrix} c_1^{\mathcal{I}} \\ s_1^{\mathcal{I}} \end{Bmatrix}_{\text{LP}} \cos\theta,$$

which are predicted by $\text{Re}/\text{Im}\mathcal{C}^{\mathcal{I}}_{\text{TP}\pm}$ and $\text{Re}/\text{Im}\mathcal{C}^{\mathcal{I}}_{\text{LP}}$.

Measurements of CFFs

Measurements of azimuthal angular coefficients

Charge combined with spin and azimuthal asymmetries allow to measure the real and imaginary part of $\mathcal{C} = \left\{ \mathcal{C}^{\mathcal{I}}_{\rm unp}, \mathcal{C}^{\mathcal{I}}_{\rm LP}, \mathcal{C}^{\mathcal{I}}_{\rm TP+}, \mathcal{C}^{\mathcal{I}}_{\rm TP-} \right\}$:

1. unpolarized beam and target $\to \mathrm{Re}\mathcal{C}^{\mathcal{I}}_{\rm unp}$
2. double spin flip with longitudinal polarized target $\to \mathrm{Re}\mathcal{C}^{\mathcal{I}}_{\rm LP}$
3. double spin flip with transverse polarized target $\to \mathrm{Re}\mathcal{C}^{\mathcal{I}}_{\rm TP\pm}$
4. polarized beam and unpolarized target $\to \mathrm{Im}\mathcal{C}^{\mathcal{I}}_{\rm unp}$
5. unpolarized beam and longitudinal polarized target $\to \mathrm{Im}\mathcal{C}^{\mathcal{I}}_{\rm LP}$
6. unpolarized beam and transverse polarized target $\to \mathrm{Im}\mathcal{C}^{\mathcal{I}}_{\rm TP\pm}$

Extraction of Compton form factors

$$\left\{ \begin{matrix} \mathrm{Re} \\ \mathrm{Im} \end{matrix} \right\} \left\{ \mathcal{C}^{\mathcal{I}}_{\rm unp}, \mathcal{C}^{\mathcal{I}}_{\rm LP}, \mathcal{C}^{\mathcal{I}}_{\rm TP+}, \mathcal{C}^{\mathcal{I}}_{\rm TP-} \right\} \to \left\{ \begin{matrix} \mathrm{Re} \\ \mathrm{Im} \end{matrix} \right\} \left\{ \mathcal{H}, \mathcal{E}, \widetilde{\mathcal{H}}, \widetilde{\mathcal{E}} \right\}.$$

Experimental consistency check

The $c_0^{\mathcal{I}}$ terms and the charge even part, i.e. $|\mathcal{T}_{\rm BH}|^2 + |\mathcal{T}_{\rm DVCS}|^2$ give a complete experimental consistency check (after subtraction of BH).

Access to skewed parton distributions

A deconvolution of the predictions is practically impossible. Parametrizations of GPDs are to compare with data (relying on LO):

$$\mathrm{Im}\mathcal{F} = \pi \sum_{i=u,d,\ldots} Q_i^2 \left[F_i(\xi, \xi, \Delta^2, \mathcal{Q}^2) - F_i(-\xi, \xi, \Delta^2, \mathcal{Q}^2) \right],$$

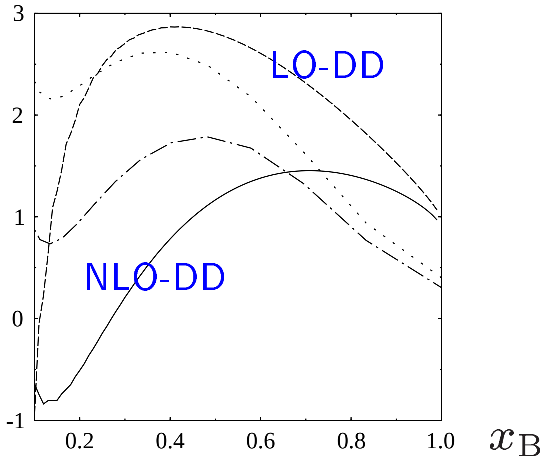
$$\mathrm{Re}\mathcal{F} = \sum_{i=u,d,\ldots} Q_i^2 \mathrm{PV} \int_{-1}^{1} dx \frac{F_i(x, \xi, \Delta^2, \mathcal{Q}^2) - F_i(-x, \xi, \Delta^2, \mathcal{Q}^2)}{\xi - x}.$$

Beyond leading order predictions

LO predictions are consistent with data from HERMES, JLAB (single spin asymmetry), H1 and ZEUS (unpolarized cross section) [M. Vanderhaeghen, P.A.M. Guichon, M. Guidal; L. Frankfurt, A. Freund, M. Strikman; A. Belitsky, D. M, L. Niedermeier, A. Schäfer; A. Freund, M. McDermott].

Perturbative corrections

Conformal symmetry gave the complete (including evolution) NLO corrections [A. Belitsky, D. M., (A. Freund)]. Their numerical size is model dependent, is moderate for the evolution and can be large for the hard scattering part (valence region)

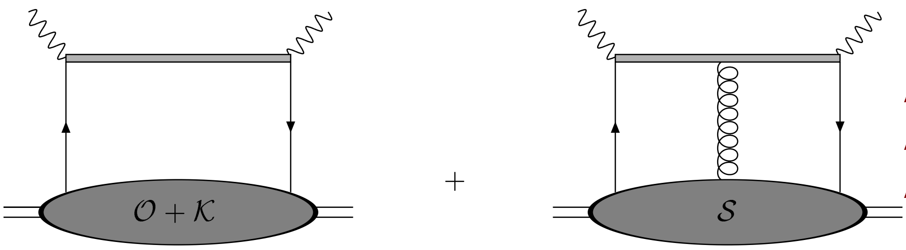


Real part of the twist-two DVCS amplitude (vector part) for the DD(FPD)-model at LO and NLO is shown as dashed (dotted) and solid (dash-dotted) lines, respectively.
A. Belitsky, D. M., L. Niedermeier, A. Schäfer;

A. Freund, M. McDermott.

Twist-three contributions

Twist-three effects for DVCS has been calculated at the Born level



I.V. Anikin, B. Pire, O.V. Teryaev;
M. Pentinen, M.V. Polyakov, A.G. Shuvaev, M. Strikman;
A.V. Belitsky, D. Müller; (complete)
A.V. Radyushkin, Ch. Weiss;
B. White.

Complete azimuthal angular dependence [A. Belitsky, A. Kirchner, D. M.].

Power suppressed contributions

Target mass corrections have been considered by [A. Belitsky, D. M.], while $O(\Delta^2/\mathcal{Q}^2)$ contributions are known for $\cos(3\phi)$ [N. Kivel, L. Mankiewicz]. The complete treatment is desired BUT complicated!

Summary and conclusions

- The electroproduction of γ in the light-cone dominated region is measurable and can be analyzed with the operator product expansion: systematic calculation of perturbative corrections and higher twist corrections.
- Theoretical predictions can be experimentally tested by:
  i. $\mathcal{Q}$ scaling of the Compton form factors.
  ii. Dominance of the leading twist-two harmonics.
  iii. Relations among coefficient functions.
- Twist-two and -three sectors are separated from each other by the azimuthal angular dependence. All Fourier coefficients are predicted by twist-two or twist-three Compton form factors, which are well defined in term of GPDs. Twist-three contains new dynamical information about quark-gluon-quark correlations.
- Access of GPDs can be done in three steps:
  i. Measurements of Compton form factors due to the combination of charge, spin, and azimuthal asymmetries.
  ii. Flavour decomposition (at least partly possible).
  iii. Comparison of GPD-models with data.
- Where the theory stands?
  i. Factorization is proved in the twist-two sector.
  ii. Complete next-to-leading order predictions for twist-two level are derived by means of conformal symmetry.
  Two codes [a slow one for $x_{\mathrm{B}} > 0.05$ and a fast one] exist.
  iii. Complete twist-three predictions at leading order will be given.
  iv. First results for power suppressed contributions are obtained.
- What is important to do?
  i. How to parametrize GPDs?
  ii. Perturbative corrections should be studied in more detail (Why they are large?, NNLO, ?resummation?).
  iii. A consistent treatment of the twist-four sector is highly desired.