# The Annihilation of Virtual Photons into Pseudoscalar Mesons

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- The photon-pion transition form factor
- Comments on the case of $\eta$, $\eta'$
- Summary

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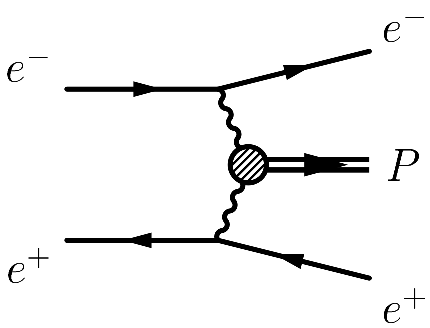
# Motivation

- Hadron distribution amplitudes (DAs): universal objects containing nonperturbative dynamics in certain exclusive hard scattering processes.
- Hadron distribution amplitudes $\longleftrightarrow$ Exclusive reactions
  Parton distribution functions $\longleftrightarrow$ Inclusive reactions.
- Meson DAs occur in elm. elastic meson form factors,
  $\gamma^{(*)} \to P$ transition form factors , $P = \pi,\ \eta,\ \eta'$,
  $\gamma^{(*)}\gamma \to \pi\pi,\ KK\ldots$
- Important ingredient in exclusive $B$-decays and, consequently, in analysis of CP violations.
- Therefore, require knowledge about shape of meson DAs.
- Question: what can be learned from simplest exclusive process, $\gamma^{(*)} \to P$ form factor, about meson DAs ?
- $\gamma^{(*)} \to P$ form factor measurable in $e^+e^- \to e^+e^-P$ at running B-factories:



# Meson distribution amplitudes

- The DA $\Phi_P$ for a pseudoscalar meson $P$ describes distribution of meson's longitudinal momentum among its (partonic) constituents.
- Valence Fock state: $\Phi_P = \Phi_P(\xi, \mu_F)$,
  $\xi = 2\,x - 1,\;\; x$ usual light-cone momentum fraction;
  $\mu_F$ factorisation scale.
- Expansion in terms of eigenfunctions of LO evolution kernel: [Lepage, Brodsky 1979]:

$$\Phi_P(\xi, \mu_F) = \Phi_{\mathrm{AS}}(\xi) \left[1 + \sum_{n=2,4,\ldots}^{\infty} B_n^P(\mu_F)\, C_n^{3/2}(\xi)\right]$$

  $C_n^{3/2}(\xi)$ : Gegenbauer polynomials

- $B_n^P$: Gegenbauer coefficients, unknown, nonperturbative quantities:

$$B_n^P(\mu_F) = B_n^P(\mu_0) \left(\frac{\alpha_s\,(\mu_F)}{\alpha_s\,(\mu_0)}\right)^{\gamma_n}$$

  $\mu_0$: typical hadronic scale ($\sim 1\,\mathrm{GeV}$)

- Asymptotic form of meson DA:

$$\Phi_{\mathrm{AS}}(\xi) = \frac{3}{2}\,(1 - \xi^2)$$

- Question (more precisely):
  do $\gamma^{(*)} \to P$ data constrain expansion coefficients $B_n^P$ ?

# The $\gamma^*$-$\pi$ transition form factor

- Leading twist, NLO $\alpha_s$-expression in hard scattering approach [Lepage, Brodsky 1980]:

$$F_{\pi\gamma^*}(\overline{Q},\omega) = \frac{1}{3\sqrt{2}}\,\frac{f_\pi}{\overline{Q}^2}\int_{-1}^{1} d\xi\,\frac{\Phi_\pi(\xi,\mu_F)}{1+\xi\omega}\left[1+\frac{\alpha_s(\mu_R)}{\pi}\,\mathcal{K}(\omega,\xi)\right]$$

- For spacelike virtualities $Q^2 = -q^2$, $Q'^2 = -q'^2$ introduce

$$\overline{Q}^2 = \frac{Q^2+Q'^2}{2}, \quad \omega = \frac{Q^2-Q'^2}{Q^2+Q'^2}$$

- Tree level diagrams:

- From asymptotic form of pion-DA follows limiting (large $Q^2$) behaviour of $F_{\pi\gamma}$ for $\omega \to \pm 1$ (real-photon limit):

$$F_{\pi\gamma}(\overline{Q},\omega=\pm 1) \to \frac{f_\pi}{\sqrt{2}\,\overline{Q}^2}$$

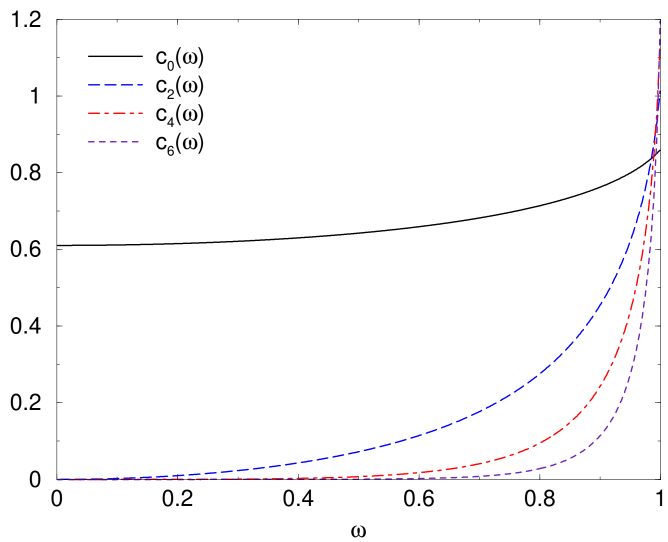
# The $\gamma^*$-$\pi$ transition form factor

- Using Gegenbauer expansion of pion-DA rewrite $F_{\pi\gamma^*}$

$$F_{\pi\gamma^*}(\overline{Q},\omega) = \frac{f_\pi}{\sqrt{2}\,\overline{Q}^2}\left[c_0(\omega,\mu_R) + \sum_{n=2,4,\dots} c_n(\omega,\mu_R,\overline{Q}/\mu_F)\,B_n^\pi(\mu_F)\right]$$

$c_n$: analytically computable functions.

- Sensitivity of $F_{\pi\gamma^*}$ w.r.t. Gegenbauer coefficients $B_n^\pi$ characterised by behaviour of $c_n$.




- Real-photon limit, $\omega \to \pm 1$:

$$c_n(\omega \to \pm 1) \to 1 + \frac{\alpha_s}{\pi}\,\mathcal{K}_n(1,\overline{Q}/\mu_F)$$

- Fast decrease of $c_n$ for $\omega \to 0$.

- $\rightsquigarrow$ Form factor sensitive to $B_n^\pi$ only for $\omega \to \pm 1$ !

# The $\gamma^*$-$\pi$ transition form factor

- In real-photon limit, $F_{\pi\gamma^*}$ approximately probes sum of Gegenbauer coefficients $B_n^\pi$:

$$F_{\pi\gamma}(\overline{Q}, \omega = \pm 1) = \frac{f_\pi}{\sqrt{2}\,\overline{Q}} \left[ 1 + \sum_{n=2,4,..} B_n^\pi(\mu_F) \right] + \mathcal{O}(\alpha_s)$$

- Constraint from CLEO data on $F_{\pi\gamma}$ [Gronberg 1998]:

$$\sum B_n^\pi \approx -0.06$$

- For informations about individual $B_n^\pi$, have to truncate sum at some $n_0$ : $B_n^\pi = 0$ for $n \geq n_0$. [Jakob, Kroll 1993]
- Theoretical and experimental uncertainties:
  choice of $\mu_R$ and $\mu_F$, power corrections,...

# The $\gamma^*$-$\pi$ transition form factor

- Power corrections through transverse momentum and Sudakov effects in endpoint regions $\xi \pm 1$:
  quark or antiquark carries small momentum fraction.

- Estimate using modified hard scattering approach:
  [Botts, Sterman 1989; Li, Sterman 1992]

$$
\begin{aligned}
F_{\pi\gamma^*}(\overline{Q},\omega) &= \frac{1}{4\sqrt{3}\pi^2}\int d\xi\, d^2\mathbf{b}\, \hat{\Psi}^*_\pi(\xi,-\mathbf{b},\mu_F) \\
&\times\; K_0(\sqrt{1+\xi\omega}\,\overline{Q}\,b)\;\exp\left[-S\left(\xi,b,\overline{Q},\mu_R\right)\right]
\end{aligned}
$$

  $\mathbf{b}$ Fourier conjugate to $\mathbf{k}_\perp$

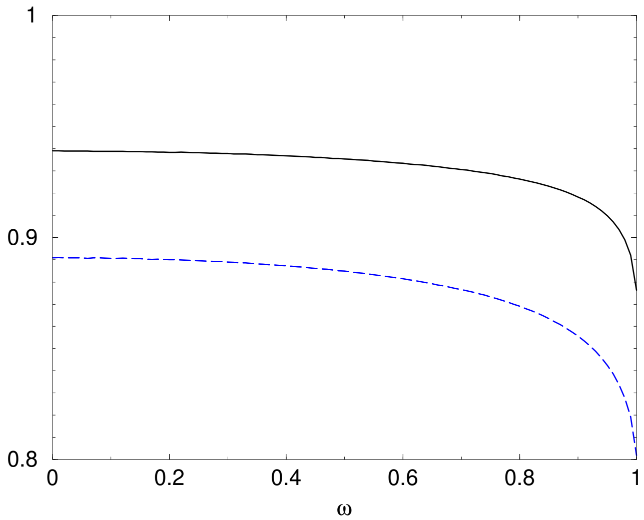
- Sudakov factor $\exp[-S]$ accounts for perturbative soft gluon corrections; suppression of large $q\bar{q}$-separations.

- Example: Gaussian ansatz for pion's light-cone wave function:
  [Jakob, Kroll 1993]

$$
\hat{\Psi}^\pi_{\mathrm{AS}}(\xi,\mathbf{b}) = \frac{2\pi f_\pi}{\sqrt{6}}\,\Phi_{\mathrm{AS}}(\xi)\,\exp\left[-\frac{\pi^2}{2}f_\pi^2\,(1-\xi^2)\,b^2\right]
$$

# The $\pi - \gamma^*$–transition form factor

- Ratio of transition form factor in modified / collinear approach



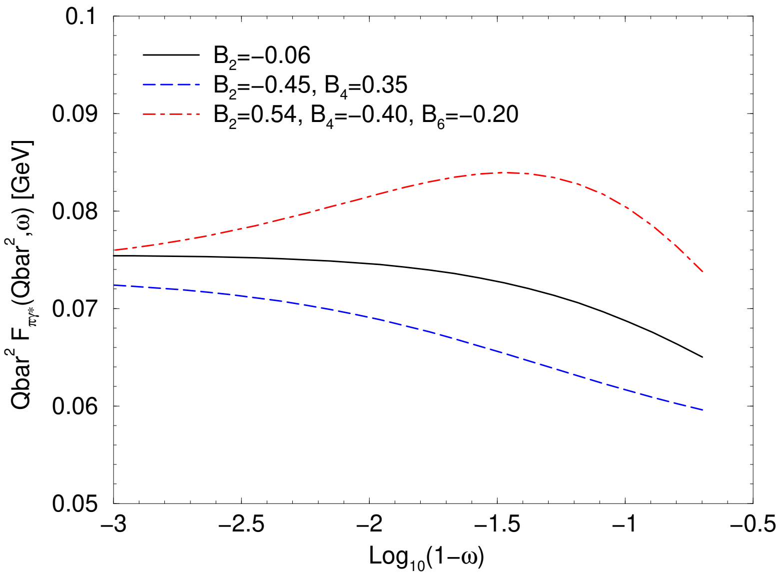

  - dashed line: $\overline{Q}^2 = 2\,\mathrm{GeV}^2$,
  - solid line: $\overline{Q}^2 = 4\,\mathrm{GeV}^2$.

- transverse momentum corrections expected to be negligible for $\overline{Q}^2 \gtrsim 4\,\mathrm{GeV}^2$ and $\omega \lesssim 0.9$: domination of leading twist.

- However, small sensitivity to coefficients $B_n^\pi$ in this region.

# The $\gamma^*$-$\pi$ transition form factor

- Region $\omega \simeq 0.9$ allows for discrimination of different theory estimates of lowest $B_n^\pi$




all $B_n^\pi$ at $\mu_0 = 1\,\mathrm{GeV}$.

# The $\gamma^*$-$\pi$ transition form factor

- Small sensitivity w.r.t. $B_n^\pi$ follows from fast decrease of $c_n(\omega)$ for $\omega \to 0$: expansion in powers of $\omega$:

$$\begin{aligned} F_{\pi\gamma^*}(\overline{Q},\omega) &= \frac{\sqrt{2} f_\pi}{3\,\overline{Q}^2}\Big[1 - \frac{\alpha_s(\overline{Q})}{\pi} + \frac{1}{5}\,\omega^2\Big(1 - \frac{5}{3}\frac{\alpha_s(\overline{Q})}{\pi}\Big) \\ &+ \frac{12}{35}\,\omega^2 B_2^\pi(\overline{Q})\Big(1 + \frac{5}{12}\frac{\alpha_s(\overline{Q})}{\pi}\Big)\Big] + \mathcal{O}(\omega^4, \alpha_s^2) \end{aligned}$$

- First appearance of $B_n^\pi$ at order $\omega^n$.

- Parameter-free prediction of pQCD to leading twist accuracy for $\omega \to 0$: [Cornwall, 1966; Köpp, Walsh, Zerwas, 1974]

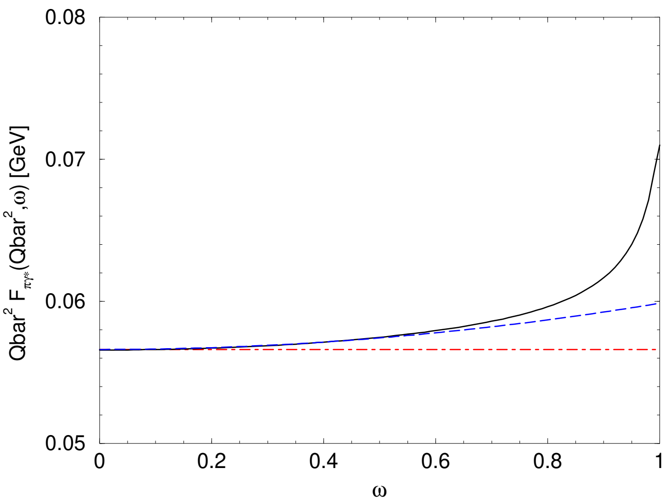
$$\overline{Q}^2 F_{\pi\gamma^*}(\overline{Q},\omega) = \frac{\sqrt{2} f_\pi}{3}\left[1 - \frac{\alpha_s(\overline{Q})}{\pi}\right] + \mathcal{O}(\omega^2, \alpha_s^2)$$

valid over a wide range of $\omega$ !

- Any deviation from this prediction would signal power corrections !

# The $\gamma^*$-$\pi$ transition form factor

- Quality of approximation of $F_{\pi\gamma^*}(\overline{Q}, \omega)$ at NLO, leading-twist for sample pion-DA ($B_2^\pi = -0.45$, $B_4^\pi = 0.35$ at $\mu_0 = 1\,\mathrm{GeV}$):

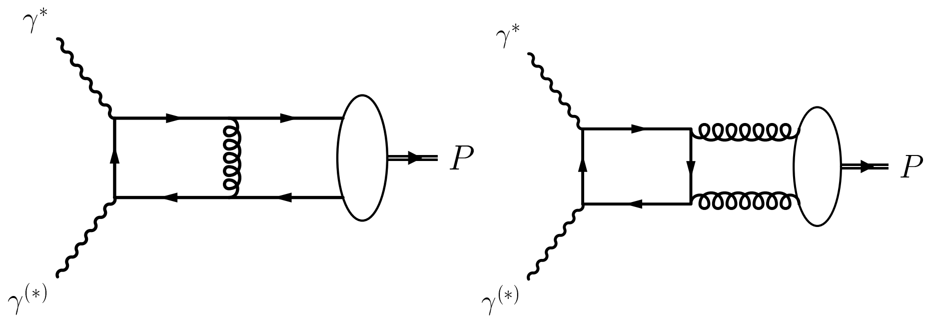



$\overline{Q}^2 = 4\,\mathrm{GeV}^2$, $\mu_F = \mu_R = \overline{Q}$

- solid line: full result
- dashed line: $\mathcal{O}(\omega^2)$
- dot-dashed line: $\mathcal{O}(\omega^0)$

# Comments on the case of eta, eta'

- Analogous discussion for $\gamma^* \to \eta,\ \eta^*$ transitions.
- Complications:
  - $\eta - \eta'$ mixing
  - contributions from gluon DA through evolution and at $\mathcal{O}(\alpha_s)$.




- Same conclusion as in pion case:
  small sensitivity to Gegenbauer expansion coefficients.

# Summary

- Meson DAs important, universal objects which embody soft physics in hard exclusive reactions.
- Analysis of photon-meson transition form factor using Gegenbauer expansion of meson DA.
- Discussion of photon-meson transition form factor falls into two kinematical regimes:
  - real-photon limit and $Q'^2 \ll Q^2$:
    $\gamma \to P$ transition probes sum of expansion coefficients $B_n^\pi$;
    for $Q'^2 \ll Q^2$ (restricted) sensitivity to size of $B_n^\pi$.
  - region $Q'^2 \sim Q^2$:
    $\gamma^* \to P$ independent of shape of meson DA
    $\rightsquigarrow$ parameter-free leading twist prediction of pQCD at moderately large $Q^2$, $Q'^2$
    deserves experimental verification !
- Running $B$-factories Barbar, Belle and CLEO appear most suitable for experimental analysis.